Plasma Measurement I

Langmuir Probes

Double Probes

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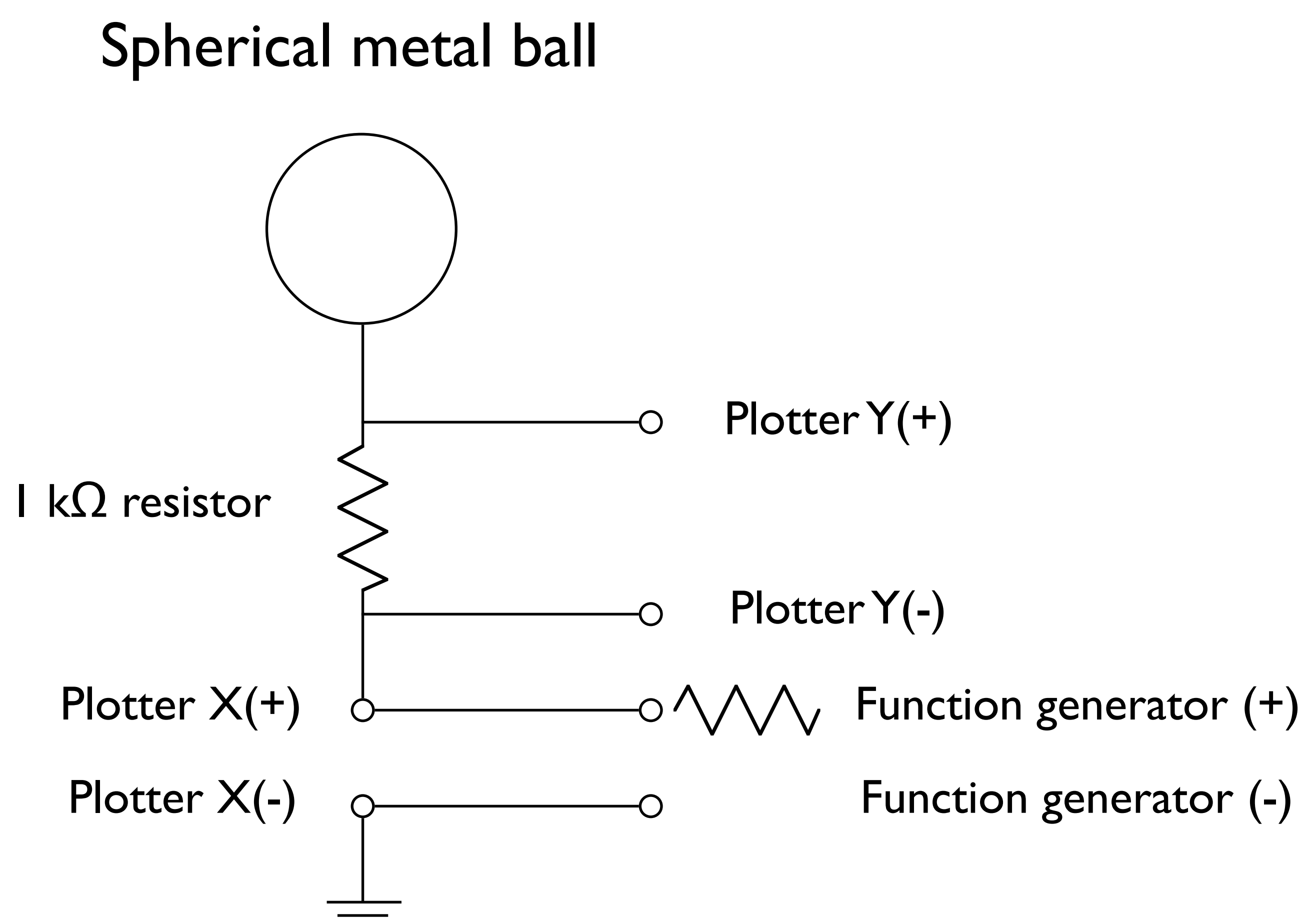
Outline

- Limitations of single probes
- The characteristics of double probes
- Spacecraft charging and its effect on in-situ plasma measurement
- Electron temperature probe
- Note: This lecture note was extracted from the following books:

Lochte-Holtgreven, W., *Plasma Diagnostics*, North-Holland Publishing Company, Amsterdam, 1968,

Huddlestone, R. H. and S. L. Leonard, *Plasma Diagnostic Techniques*, 1968

Circuit of Langmuir probe



Limitations of single probes

In most gas discharge there is an electrode in good contact with plasma which can be used as **a reference point for potential when applying a bias voltage of a probe**.

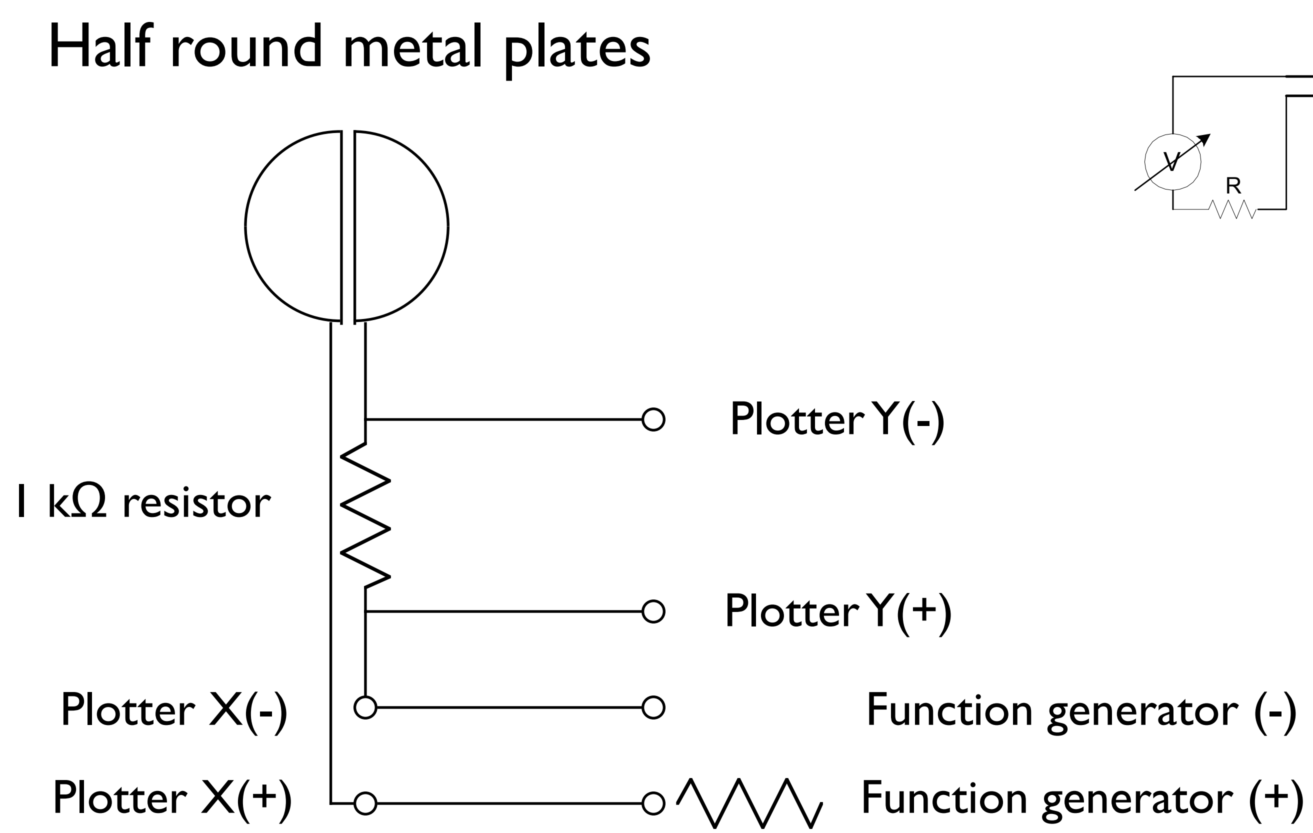
Such an electrode can be the anode or cathode of a discharge, or the **metallic wall** or limiter of an electrode-less discharge, such as that in a stellarator or toroidal pinch.

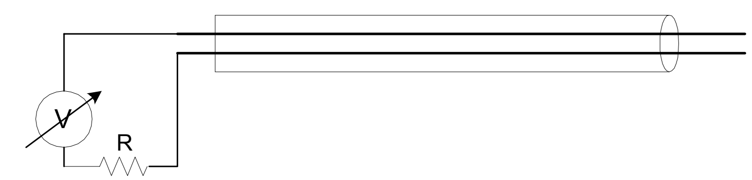
Limitations of single probes (cont.)

In some instances such a reference point is not available. Examples of this are a toroidal RF discharge in a glass tube or **the plasma in the ionosphere**.

The single probe method fails in many experimental situations, particularly when **a large reference electrode is absent** or **when the space potential is not well defined**. In decaying plasma, in which the plasma potential changed with time, so that it was difficult to maintain a constant probe-plasma potential difference.

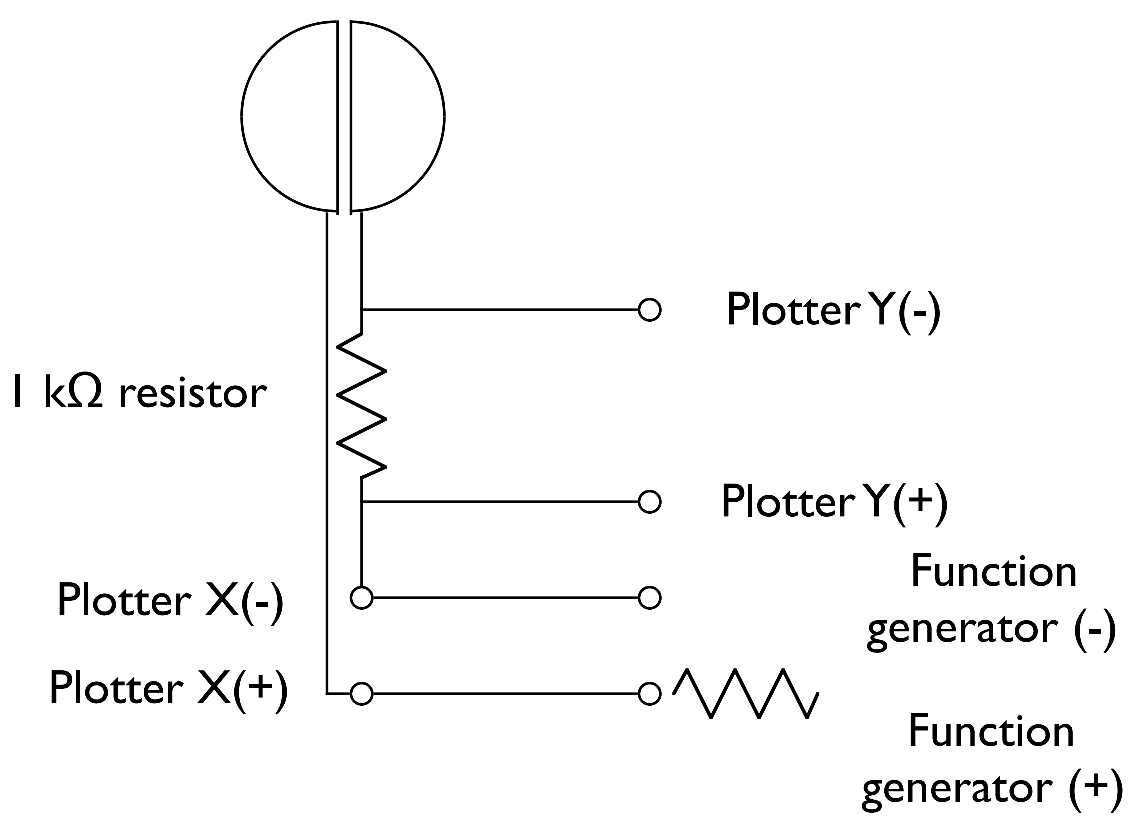
Circuit of double probe



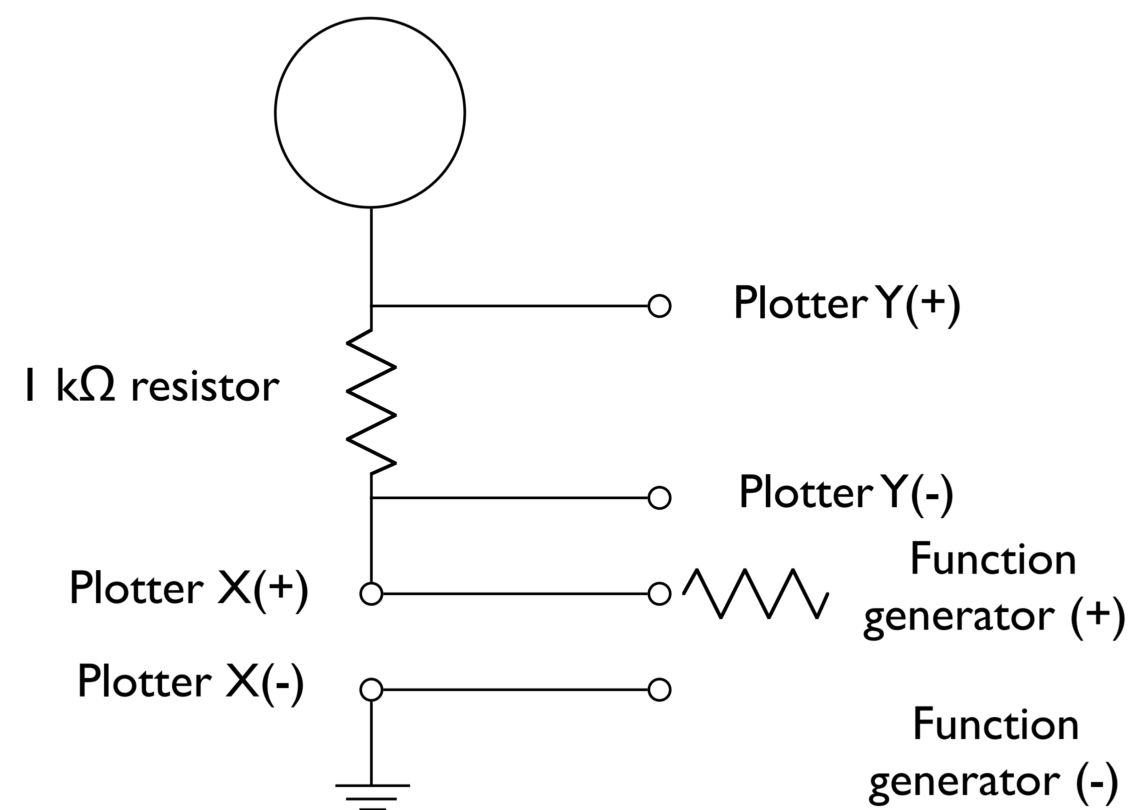


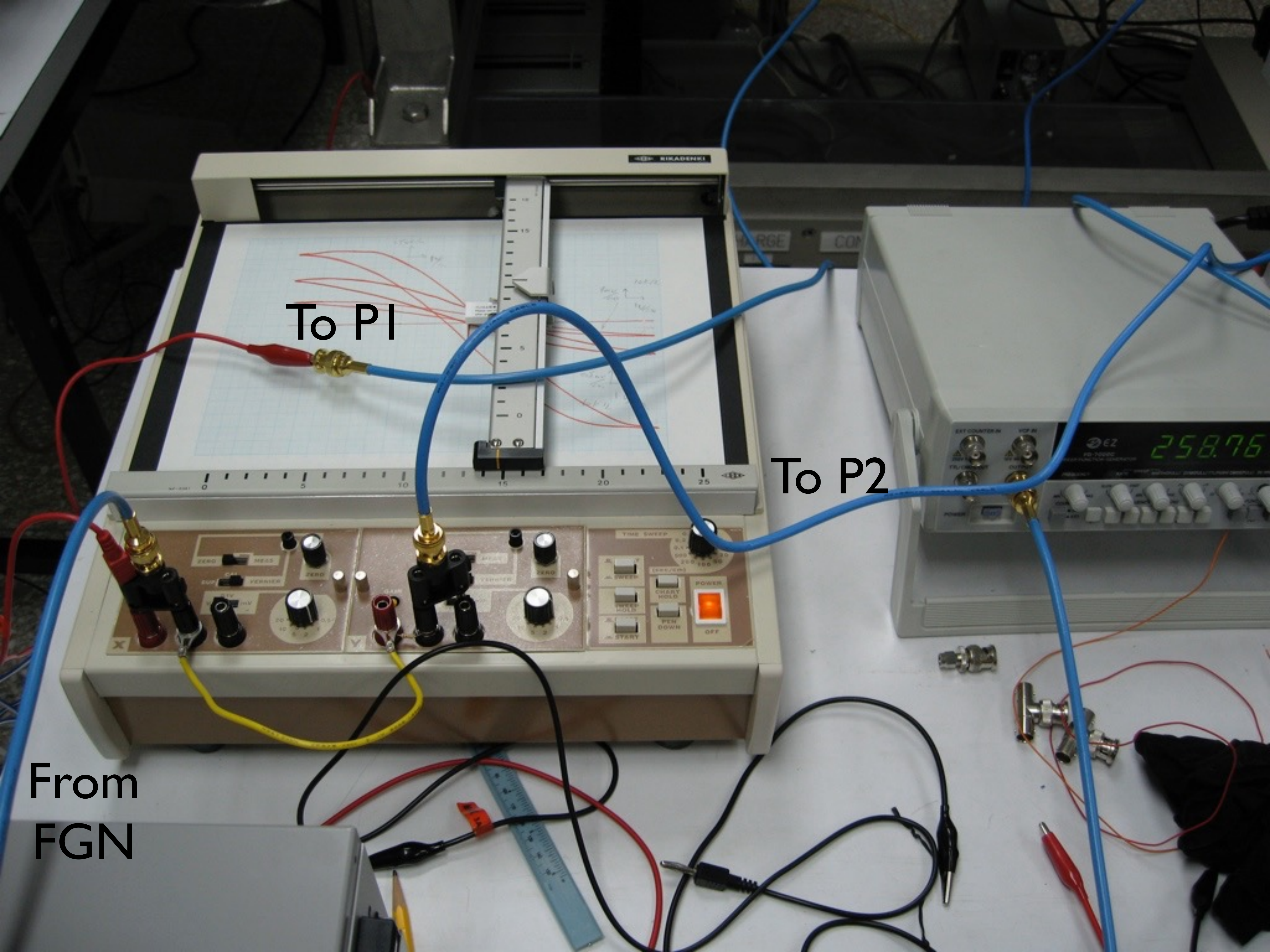
Comparisons

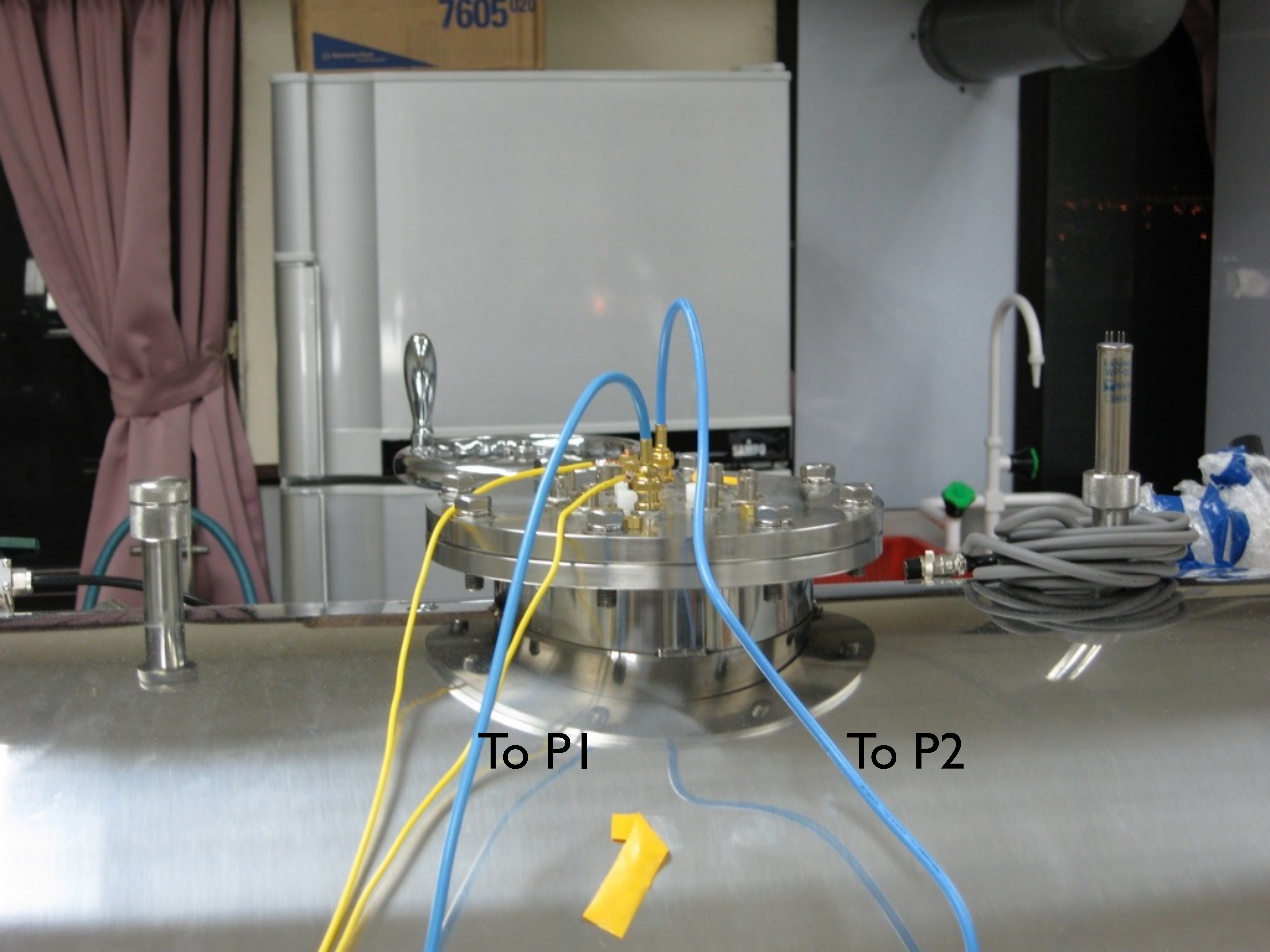
Half round metal plates

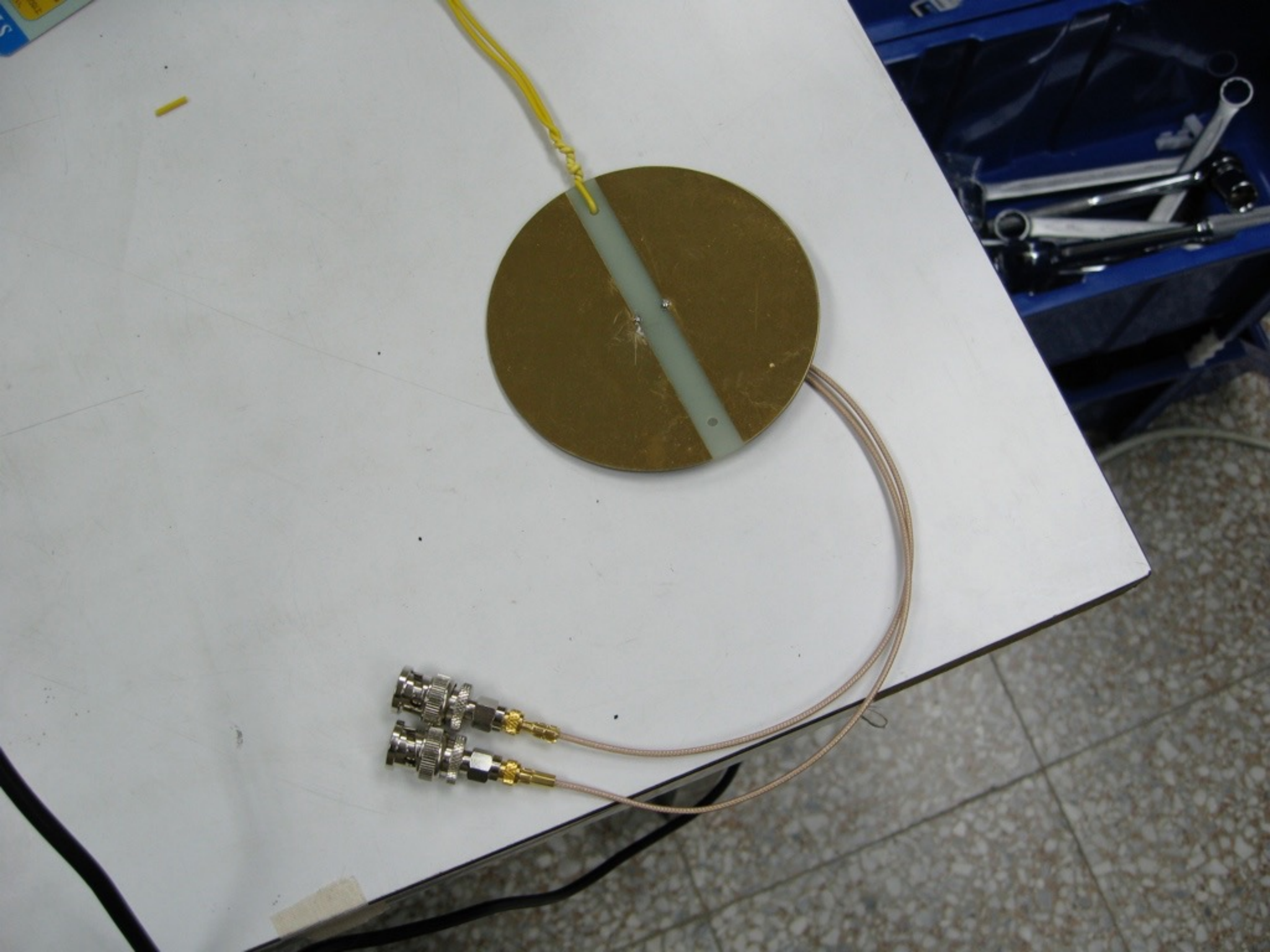


Spherical metal ball

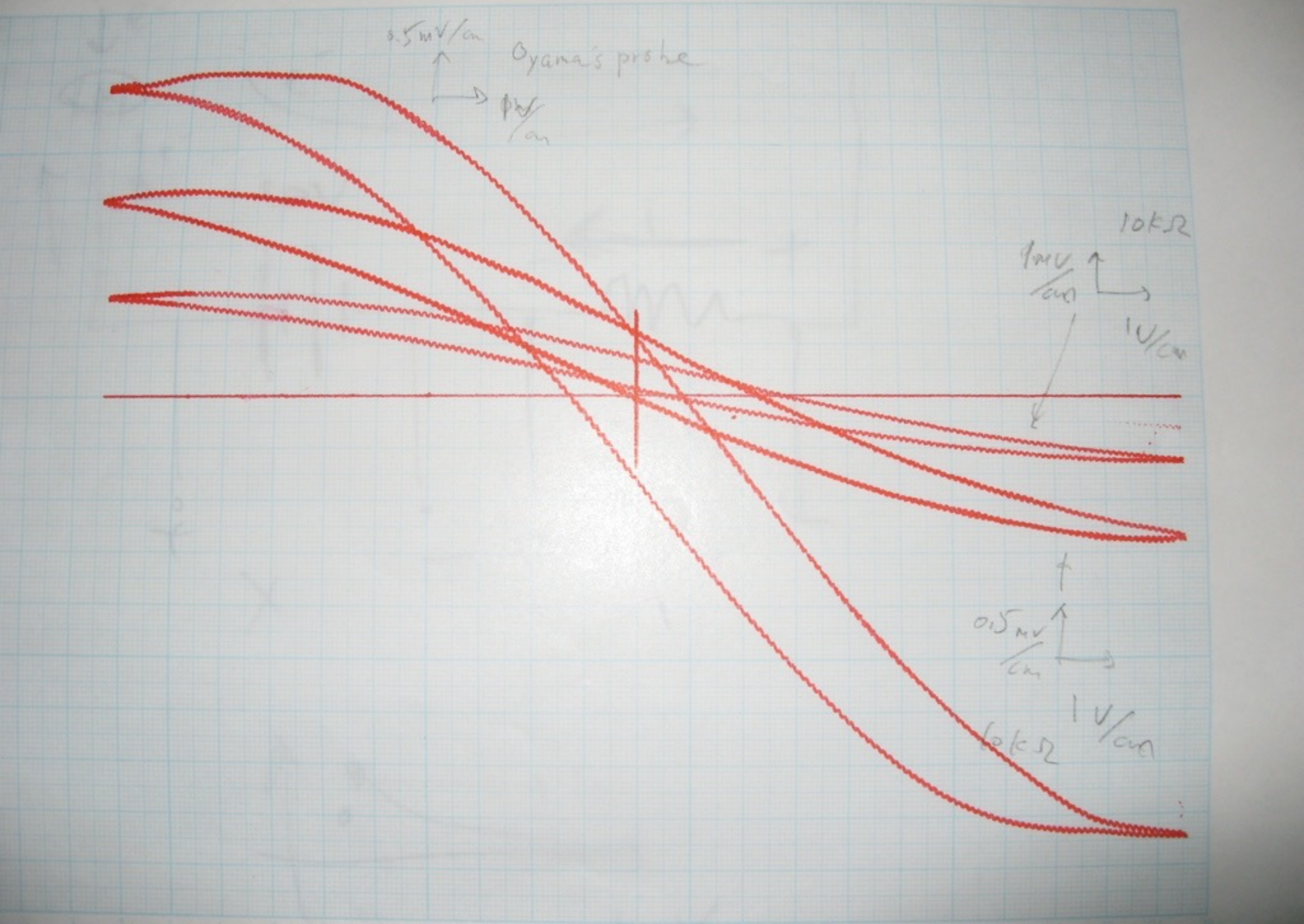



To P1
To P2
From FGN
RIKADENKI
TIME SWEEP
POWER
OFF


7605
To P1
To P2



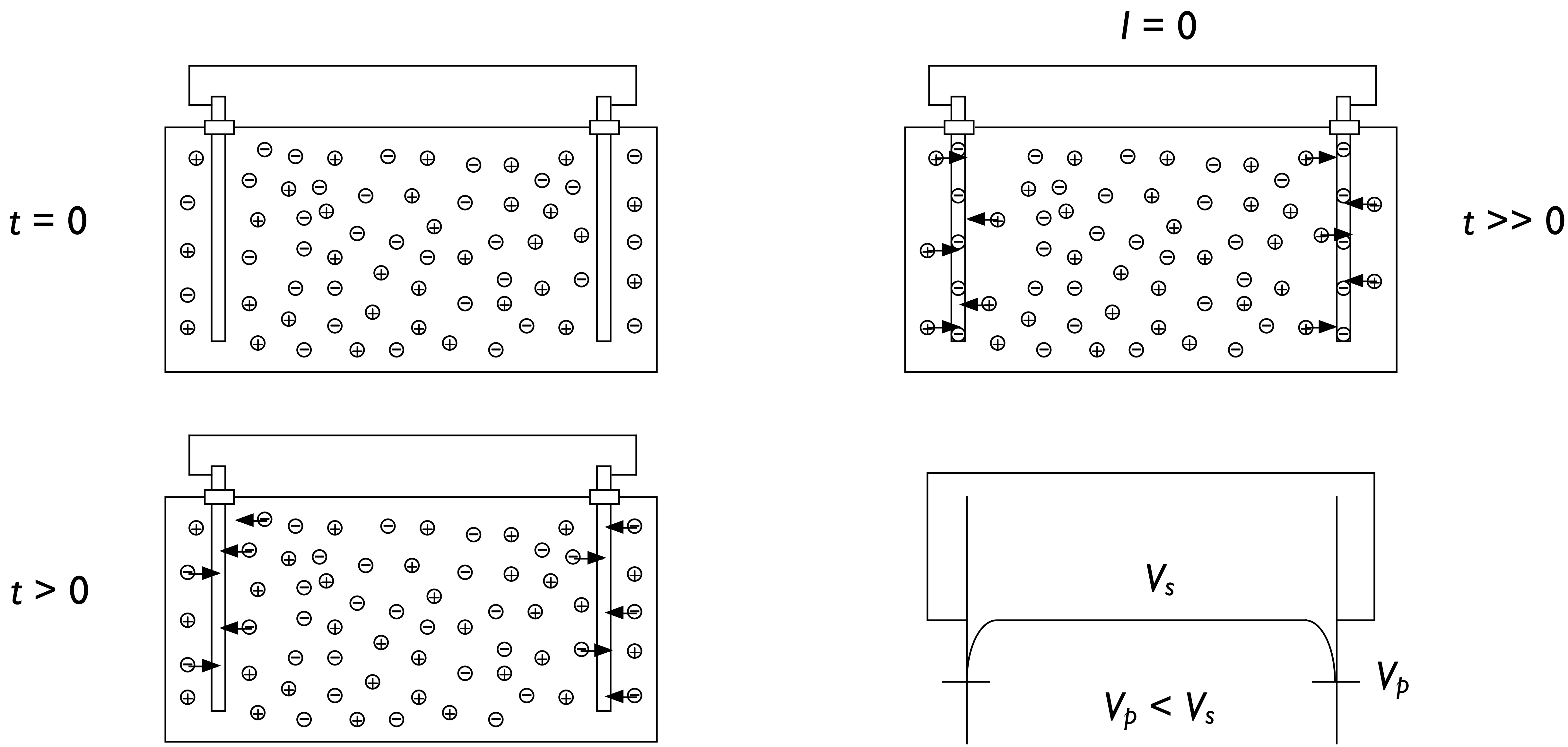
double probe



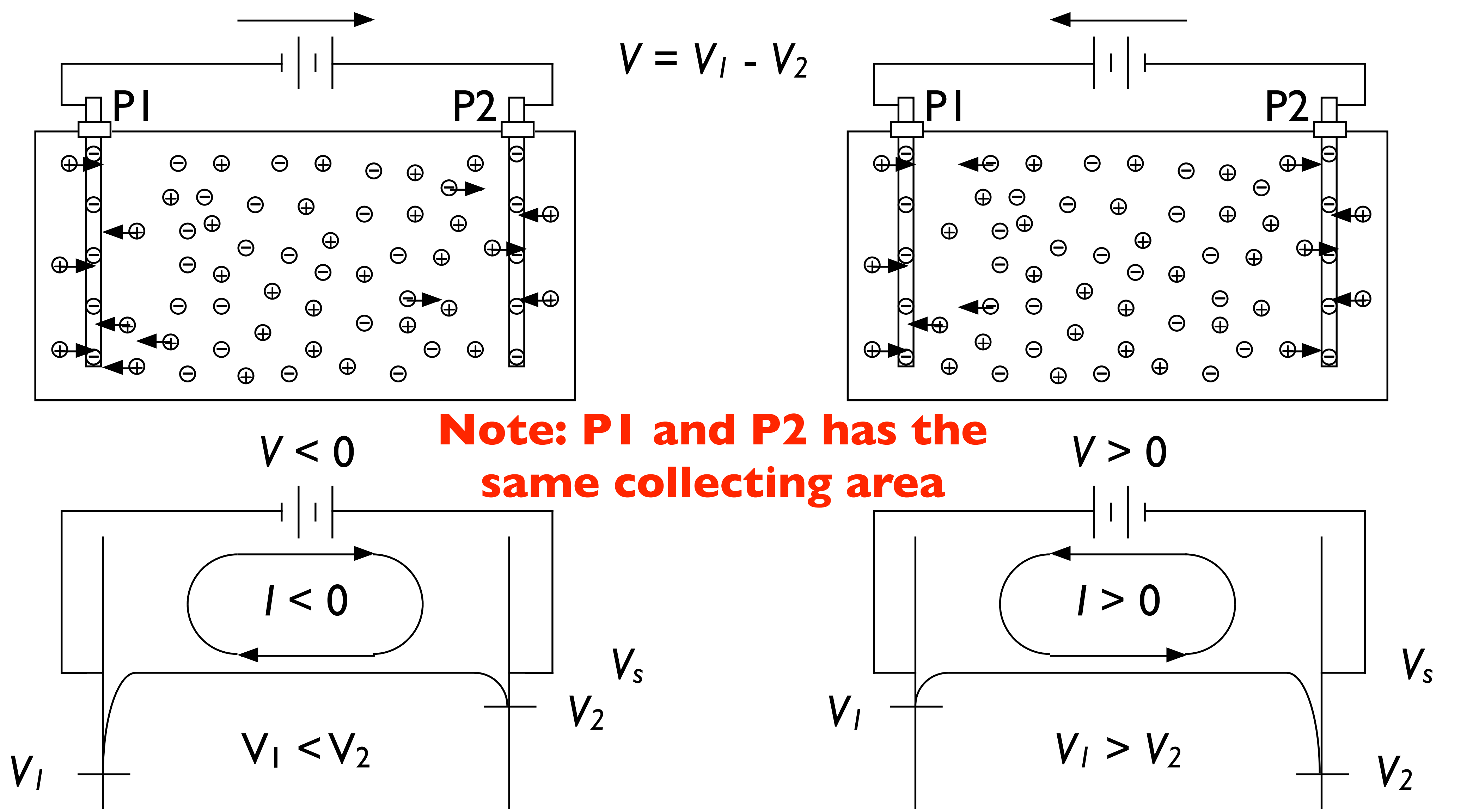
Double probes

The double probe, which was first proposed and utilized by **Johnson and Malter (1950)** partly obviates these disadvantages of Langmuir probes. With two probes biased with respect to each other but insulated from ground, **the entire system "floats" with the plasma and therefore follows the change of plasma potential**. Since the electron velocities are much higher than the ion velocities, the probe in general must both be negative with respect to space to prevent a net electron current from flowing to the whole system. This condition can be violated only if one probe is much larger than the other that the ion current to the larger probe can cancel the electron saturation current to the smaller probe. The total current to the system can never be greater than the **ion saturation current collected by the larger probe**.

Two connected electrodes floating with plasma



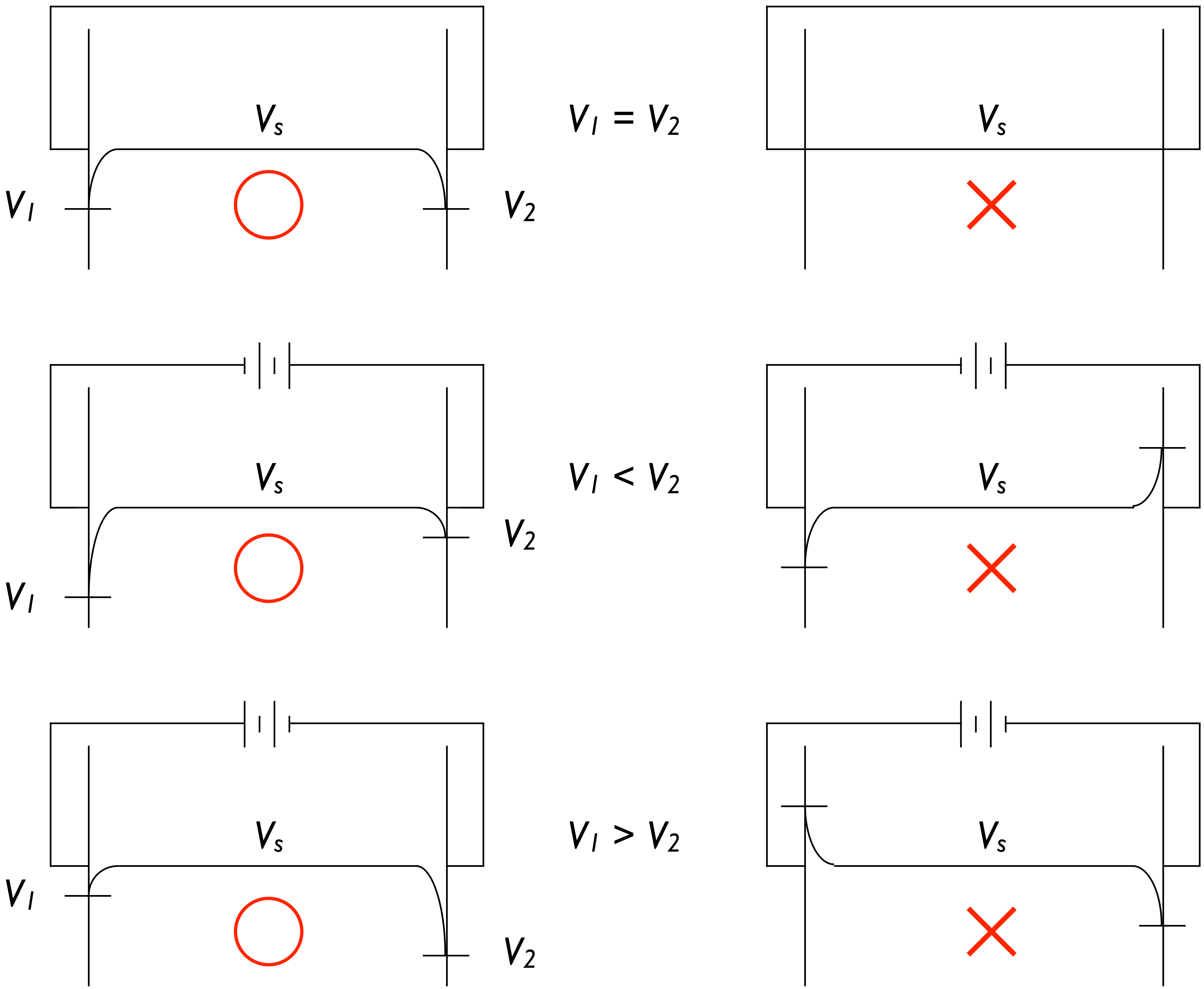
Applying a voltage between two electrodes



More electrons will flow to probe 2 and fewer to probe 1. It results in a positive current flow from 2 to 1 in plasma and from 1 to 2 in wire connection.

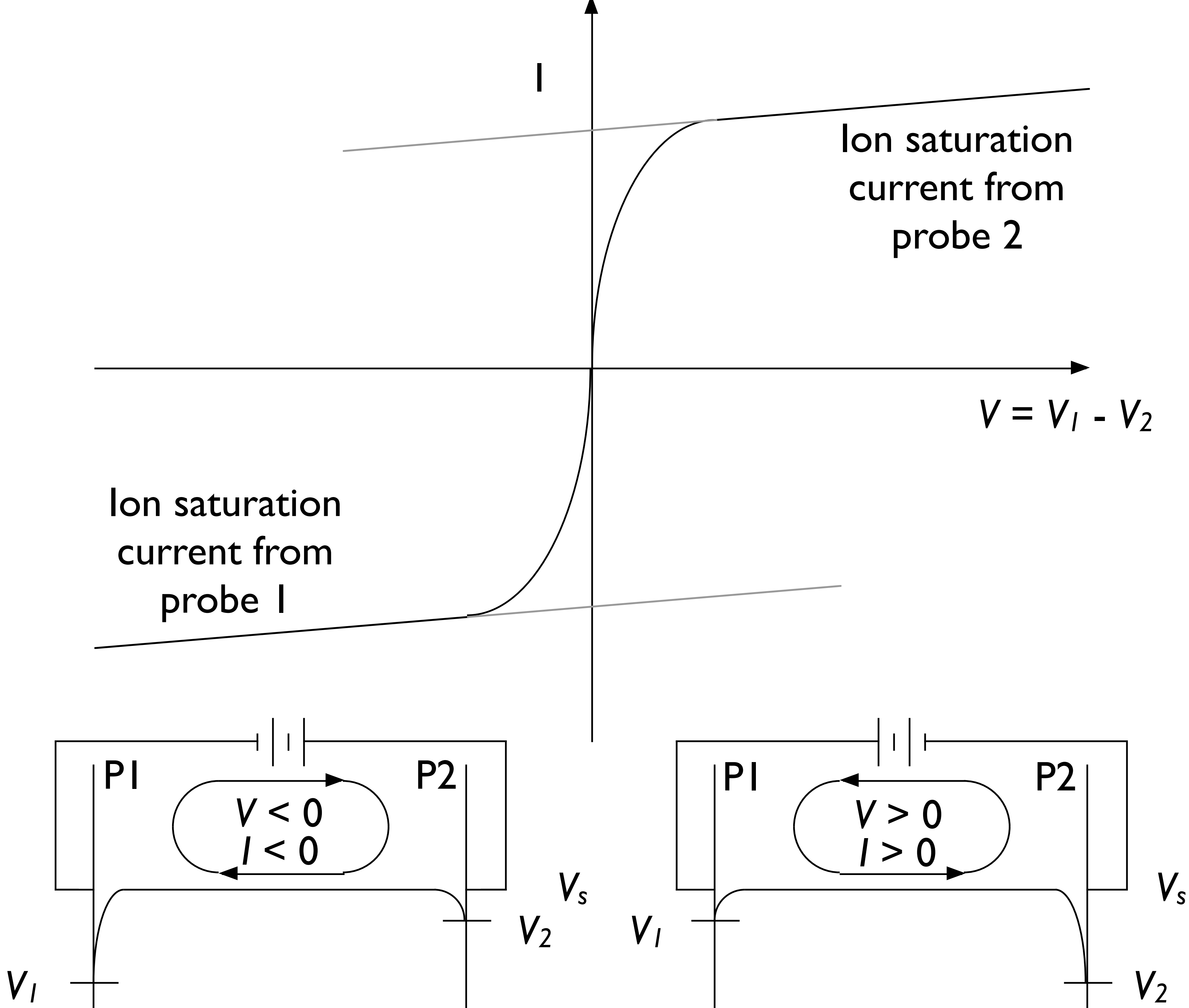
More electrons will flow to probe 1 and fewer to probe 2. It results in a positive current flow from 1 to 2 in plasma and from 2 to 1 in wire connection.

Note: P1 and P2 has the same collecting area



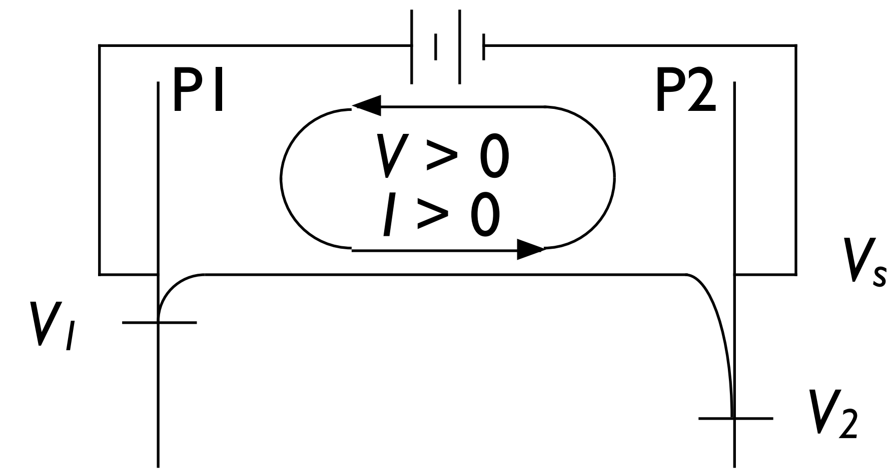
$$\frac{\Phi_{es}}{\Phi_{O^+s}} \sim \sqrt{\frac{M_i}{m_e}} \sim \sqrt{1837 \times 16} \sim 171$$

Why?



The total current to the system can never be greater than the **ion saturation current**, since any electron current to the total system must always be balanced by an equal ion current. However, only the fast electrons in the tail of the distribution can ever be collected; the bulk of the electron distribution is not sampled.

To find the current quantitatively, we define the I_{+1}, I_{e1}, I_{+2}, and I_{e2} to be the ion and electron currents to probe 1 and 2 at any given V and are all positive. The current I (can be positive or negative) in the loop is given by



$$I = I_{e1} - I_{+1} = I_{+2} - I_{e2} \rightarrow \frac{I_{e1}}{I_{e2}} = \frac{I + I_{+1}}{I_{+2} - I}$$

The current I_{e1} and I_{e2} are given for the electron currents to a probe in the transition region:

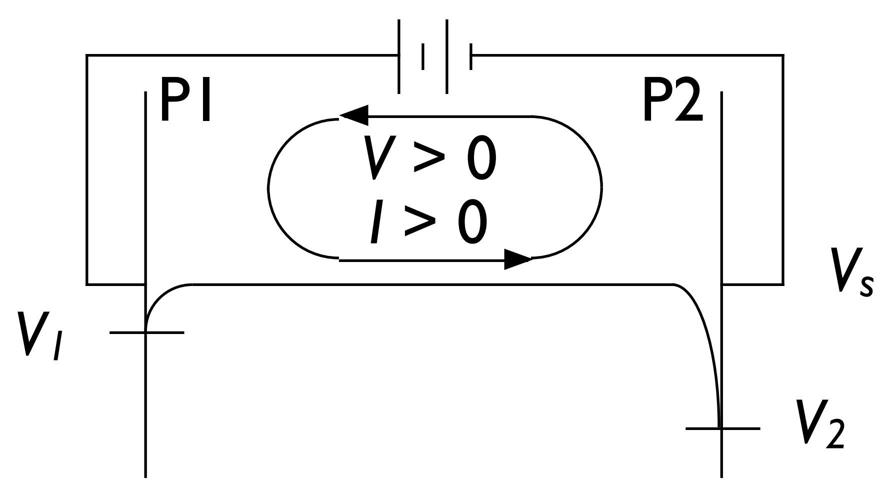
$$I_{e1} = A_1 e n_e \upsilon_{T_e} \exp\left(\frac{eV_1}{\kappa T_e}\right) \text{ and } I_{e2} = A_2 e n_e \upsilon_{T_e} \exp\left(\frac{eV_2}{\kappa T_e}\right)$$

where the V_1 and V_2 are referenced to V_s (i. e. V_s should be assumed to be zero).

The equation can be re-organized as

$$\frac{I_{e1}}{I_{e2}} = \frac{A_1 e n_e \upsilon_{T_e} \exp\left(\dfrac{eV_1}{\kappa T_e}\right)}{A_2 e n_e \upsilon_{T_e} \exp\left(\dfrac{eV_2}{\kappa T_e}\right)} = \frac{A_1}{A_2}\exp\left(\frac{eV}{\kappa T_e}\right) = \frac{I + I_{+1}}{I_{+2} - I}$$

where $I \rightarrow I_{+2}$, if $V >> 0$ and $I \rightarrow -I_{+1}$, if $V << 0$. The basic assumption of this theory is that the probes are always negative enough to be collecting essentially ion saturation current; therefore, ion saturation current can be accurately estimated at any V by smoothly extrapolating the saturation portions of the double-probe characteristic.



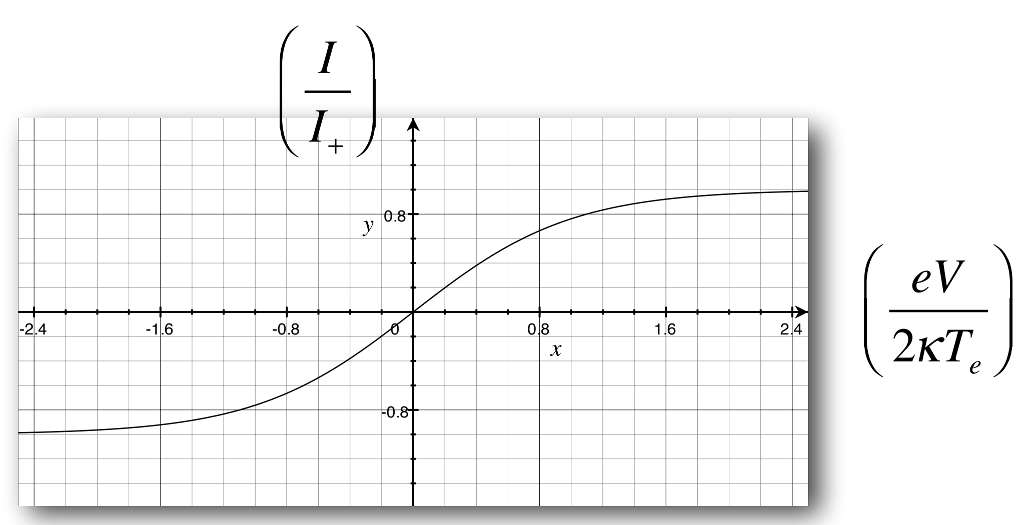
If $A_1 = A_2$, then $I_{+1} = I_{+2} = I_+$, and I can be solved as

$$I = I_+ \tanh\left(\frac{eV}{2\kappa T_e}\right)$$

where

$$I + I_+ = (I_+ - I)\exp\left(\frac{eV}{\kappa T_e}\right) \rightarrow \left[\exp\left(\frac{eV}{\kappa T_e}\right) + 1\right] I = \left[\exp\left(\frac{eV}{\kappa T_e}\right) - 1\right] I_+$$

$$\rightarrow I = I_+ \frac{\exp\left(\frac{eV}{\kappa T_e}\right) - 1}{\exp\left(\frac{eV}{\kappa T_e}\right) + 1} = I_+ \frac{\exp\left(\frac{eV}{2\kappa T_e}\right) - \exp\left(\frac{-eV}{2\kappa T_e}\right)}{\exp\left(\frac{eV}{2\kappa T_e}\right) + \exp\left(\frac{-eV}{2\kappa T_e}\right)} = I_+ \tanh\left(\frac{eV}{2\kappa T_e}\right)$$



If $A_1 >> A_2$, we can assume that probe 1 is essentially unaffected by probe 2 and is almost at floating potential, with then $I_{+1} \sim I_{e1}$ and $|I| << I_{+1}$. Thus

$$\frac{I_{e1}}{I_{e2}} = \frac{I + I_{+1}}{I_{+2} - I} \sim \frac{I_{+1}}{I_{e2}} = \frac{A_1}{A_2} \exp\left(\frac{eV}{\kappa T_e}\right) = \frac{A_1}{A_2} \exp\left[\frac{e(V_1 - V_2)}{\kappa T_e}\right]$$

$$\rightarrow I_{e2} = \frac{A_2}{A_1} I_{+1} \exp\left(\frac{-eV}{\kappa T_e}\right) = \frac{A_2}{A_1} I_{+1} \exp\left[\frac{-e(V_1 - V_2)}{\kappa T_e}\right]$$

Since $I_{+1} \sim I_{e1}$, the I_{+1} can be replaced by $I_{+1} \sim I_{e1} = A_1 e n_e \upsilon_{T_e} \exp\left(\frac{eV_1}{\kappa T_e}\right)$

$$I \sim -I_{e2} = -\frac{A_2}{A_1} I_{+1} \exp\left[\frac{-e(V_1 - V_2)}{\kappa T_e}\right] \sim -A_2 e n_e \upsilon_{T_e} \exp\left(\frac{eV_2}{\kappa T_e}\right)$$

This is just the transition current to a single probe for probe 2 (if V_2 is less than V_s), since probe 1 has become a large reference electrode. This case of $A_1 >> A_2$ has applications in space physics, where a nose cone casing often serves as a large reference probe.

Since only a few electrons are sampled anyway (V_1 or V_2 are less than V_s, low energy electrons are repelled away from probes), sufficient accuracy on kT_e can be obtained by merely measuring the slope of the characteristic at the origin (V_1 and V_2 are close to floating potential). If we assume that I_{+1} is independent of V, we can obtain

$$\frac{dI}{dV}=\frac{d\left(I_{e1}-I_{+1}\right)}{dV}\sim\frac{dI_{e1}}{dV}=\frac{d}{dV}\left[A_1 e n_e \upsilon_{T_e}\exp\left(\frac{eV_1}{\kappa T_e}\right)\right]$$

$$=A_1 e n_e \upsilon_{T_e}\exp\left(\frac{eV_1}{\kappa T_e}\right)\frac{e}{\kappa T_e}\frac{dV_1}{dV}$$

$$\frac{dI}{dV}=\frac{d\left(I_{+2}-I_{e2}\right)}{dV}\sim-\frac{dI_{e2}}{dV}=-\frac{d}{dV}\left[A_2 e n_e \upsilon_{T_e}\exp\left(\frac{eV_2}{\kappa T_e}\right)\right]$$

$$=-A_2 e n_e \upsilon_{T_e}\exp\left(\frac{eV_2}{\kappa T_e}\right)\frac{e}{\kappa T_e}\frac{dV_2}{dV}$$

Combining these two equations, we can get

$$V = V_1 - V_2$$

$$\frac{dI_{e1}}{dV} \sim -\frac{dI_{e2}}{dV} \rightarrow A_1 e n_e \upsilon_{T_e} \exp\left(\frac{eV_1}{\kappa T_e}\right)\frac{e}{\kappa T_e}\frac{dV_1}{dV}$$

$$= -A_2 e n_e \upsilon_{T_e} \exp\left(\frac{eV_2}{\kappa T_e}\right)\frac{e}{\kappa T_e}\frac{dV_2}{dV} = A_2 e n_e \upsilon_{T_e} \exp\left(\frac{eV_2}{\kappa T_e}\right)\frac{e}{\kappa T_e}\left(1 - \frac{dV_1}{dV}\right)$$

$$\frac{dV_1}{dV} = \frac{A_2 e n_e \upsilon_{T_e} \exp\left(\frac{eV_2}{\kappa T_e}\right)\frac{e}{\kappa T_e}}{A_1 e n_e \upsilon_{T_e} \exp\left(\frac{eV_1}{\kappa T_e}\right)\frac{e}{\kappa T_e} + A_2 e n_e \upsilon_{T_e} \exp\left(\frac{eV_2}{\kappa T_e}\right)\frac{e}{\kappa T_e}}$$

$$= \frac{\exp\left(\frac{eV_2}{\kappa T_e}\right)A_2}{\exp\left(\frac{eV_1}{\kappa T_e}\right)A_1 + \exp\left(\frac{eV_2}{\kappa T_e}\right)A_2}, \quad \frac{dV_2}{dV} = \frac{-\exp\left(\frac{eV_1}{\kappa T_e}\right)A_1}{\exp\left(\frac{eV_1}{\kappa T_e}\right)A_1 + \exp\left(\frac{eV_2}{\kappa T_e}\right)A_2}$$

For $V = 0$, it means $V_1 = V_2 = V_f$, where V_f is floating potential of the plasma (shall be less than V_s for most of cases), the I-V curve slope at $V = 0$ can be expressed as

$$\left.\frac{dI}{dV}\right|_{V=0} \sim \frac{dI_{e1}}{dV} \sim A_1 e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right)\frac{e}{\kappa T_e}\left.\frac{dV_1}{dV}\right|_{V_1=V_f}$$

$$= e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right)\frac{e}{\kappa T_e}\frac{A_1 A_2}{A_1 + A_2}$$

$$\left.\frac{dI}{dV}\right|_{V=0} \sim -\frac{dI_{e2}}{dV} \sim -A_2 e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right)\frac{e}{\kappa T_e}\left.\frac{dV_2}{dV}\right|_{V_2=V_f}$$

$$= e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right)\frac{e}{\kappa T_e}\frac{A_1 A_2}{A_1 + A_2}$$

Since $I = I_{e1} - I_{+1} = I_{+2} - I_{e2} = 0$ at $V = 0$, the I_{+1} and I_{+2} can be written as

$$I_{+1}\big|_{V_1=V_f} = I_{e1}\big|_{V_1=V_f} = A_1 e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right)$$

$$I_{+2}\big|_{V_2=V_f} = I_{e2}\big|_{V_2=V_f} = A_2 e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right)$$

The slope of I-V curve can be rewritten as

$$\left.\frac{dI}{dV}\right|_{V=0} = \frac{e}{\kappa T_e} \frac{e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right) A_1 e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right) A_2}{e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right) A_1 + e n_e \upsilon_{T_e} \exp\left(\frac{eV_f}{\kappa T_e}\right) A_2}$$

$$= \frac{e}{\kappa T_e} \frac{I_{+1}\big|_{V_1=V_f} \, I_{+2}\big|_{V_2=V_f}}{I_{+1}\big|_{V_1=V_f} + I_{+2}\big|_{V_2=V_f}}$$

Because we assume that I_{+1} and I_{+2} are independent of V, we can obtain

$$\left.\frac{dI}{dV}\right|_{V=0} = \frac{e}{\kappa T_e}\frac{I_{+1}I_{+2}}{I_{+1}+I_{+2}} \rightarrow T_e = \frac{e}{\kappa}\frac{I_{+1}I_{+2}}{I_{+1}+I_{+2}}\left[\left.\frac{dI}{dV}\right|_{V=0}\right]^{-1}$$

where $R \equiv \left[\left.\frac{dI}{dV}\right|_{V=0}\right]^{-1}$ is called **equivalent resistance**.

From this, kT_e can be computed from the I-V curve slope at the origin and the measured magnitude of I_{+1} for $V \ll 0$ and I_{+2} for $V \gg 0$.

In order to minimize the errors involved in this method (**equivalent resistance method**), it is desirable that the ion current depends as weakly as possible with V. Spherical probes for which the ion saturation current is poor should not be used in double probe arrangements. Once the kT_e is known, the plasma density can be calculated from either saturation current, with the help of the theories of ion collection.

Outline

- Requirements
- Solar panel
- Principle of in-situ plasma measurement
- Possible solutions

IVM ICD to IRD

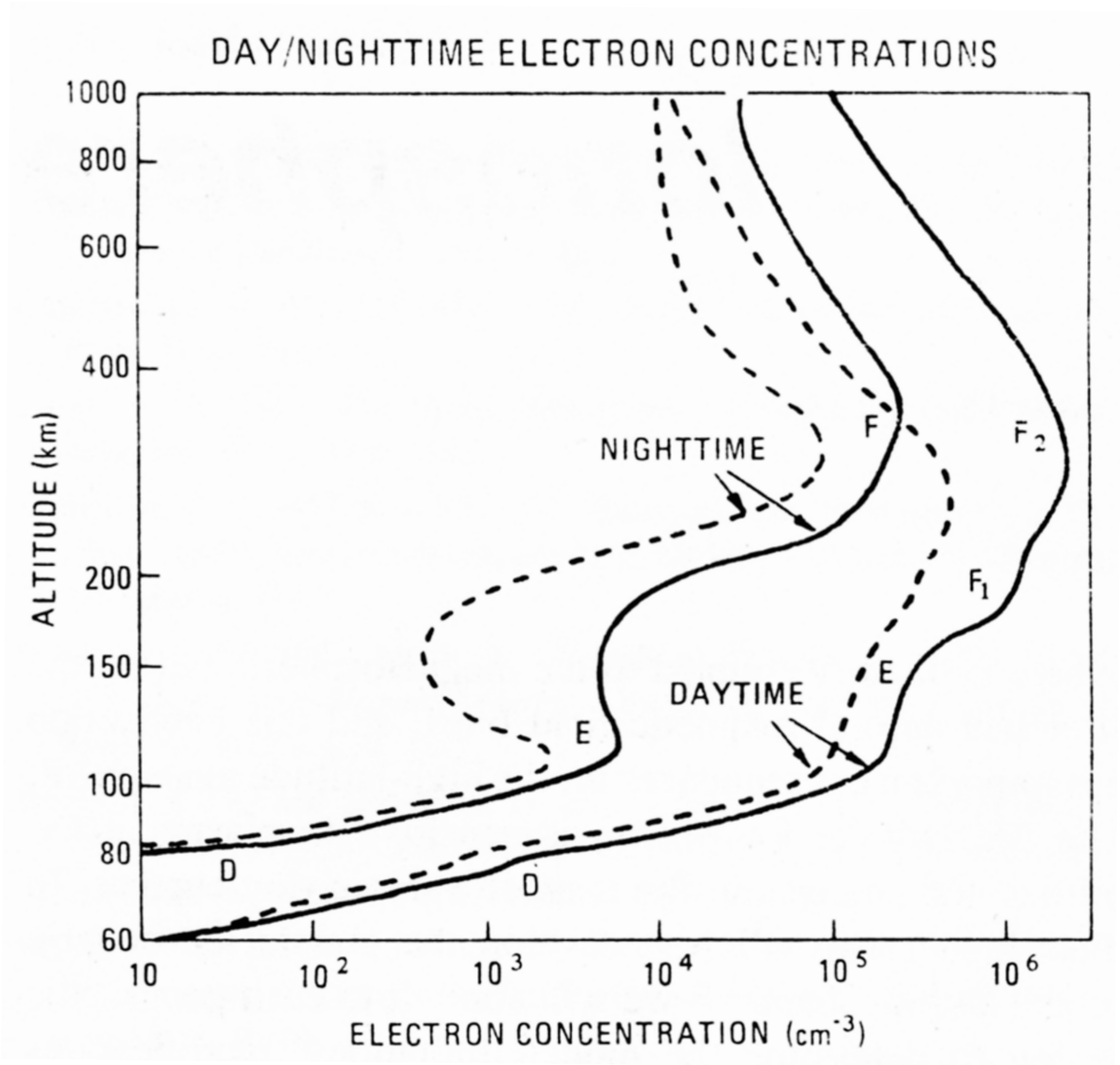
- IVMICD.00590:
  - **The spacecraft shall have no exposed potential greater than 40 V.**
  - There shall be no exposed potential on the spacecraft greater than 40 V.
- IVMICD.00600:
  - **The ram surface of the S/C shall be connected to S/C ground.**
  - No S/C surfaces having potentials in excess of ±30 V relative to spacecraft ground within 35 cm/53 cm (threshold/objective) and 45° to 90° of aperture plate edges.

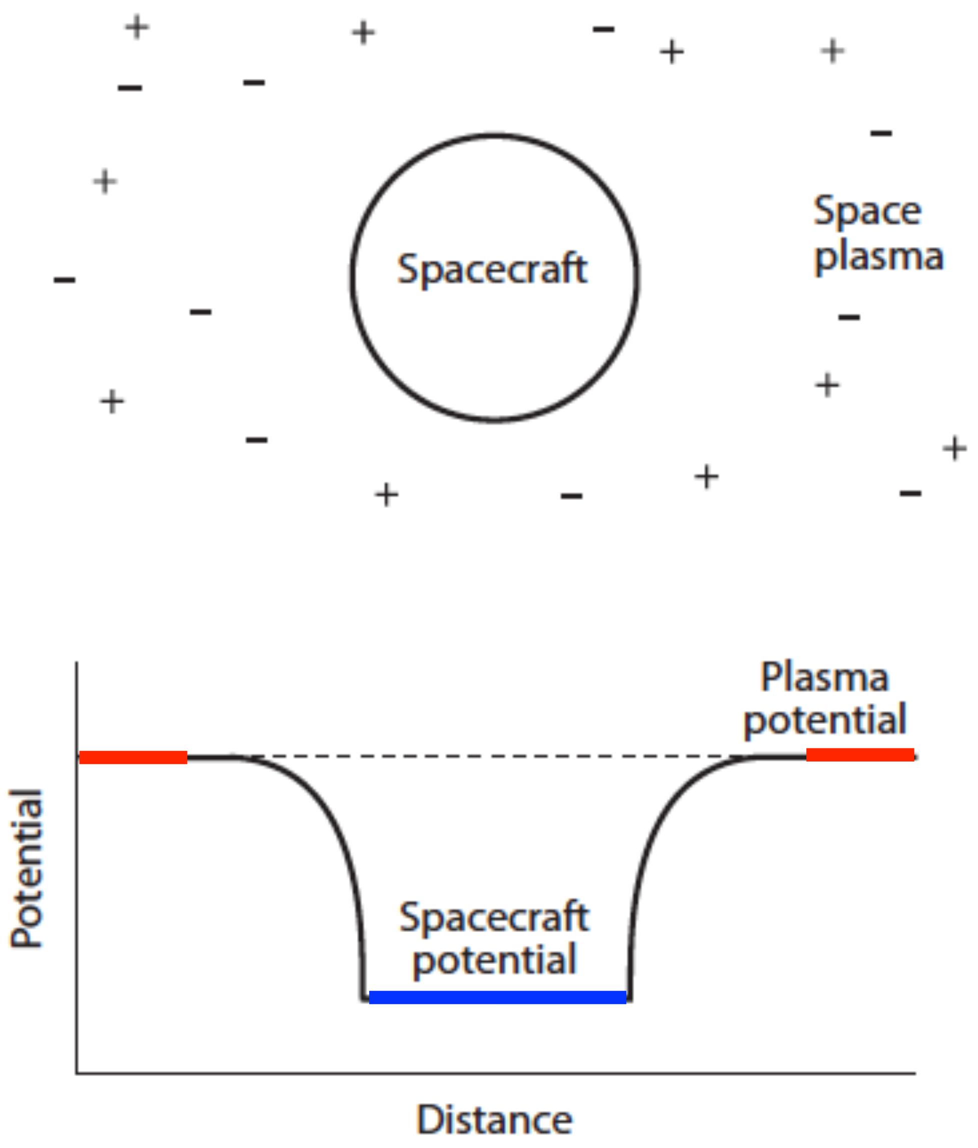
Definitions

- **Space potential** (——): Also known as the plasma potential, this refers to the electric potential within a plasma in the absence of any probes. The space potential is typically more or less uniform outside of plasma sheath regions.
- **Floating potential** (——): The potential is measured at a probe (uniform charging, only one potential for a probe) placed inside the plasma. This is because the faster electron speeds in a plasma cause a net electron current to deposit onto a floating probe until the floating probe becomes sufficiently negatively charged to repel electrons and attract ions. The result is that the floating potential is less than the actual space potential. The net current to the probe will be zero in a steady state condition.

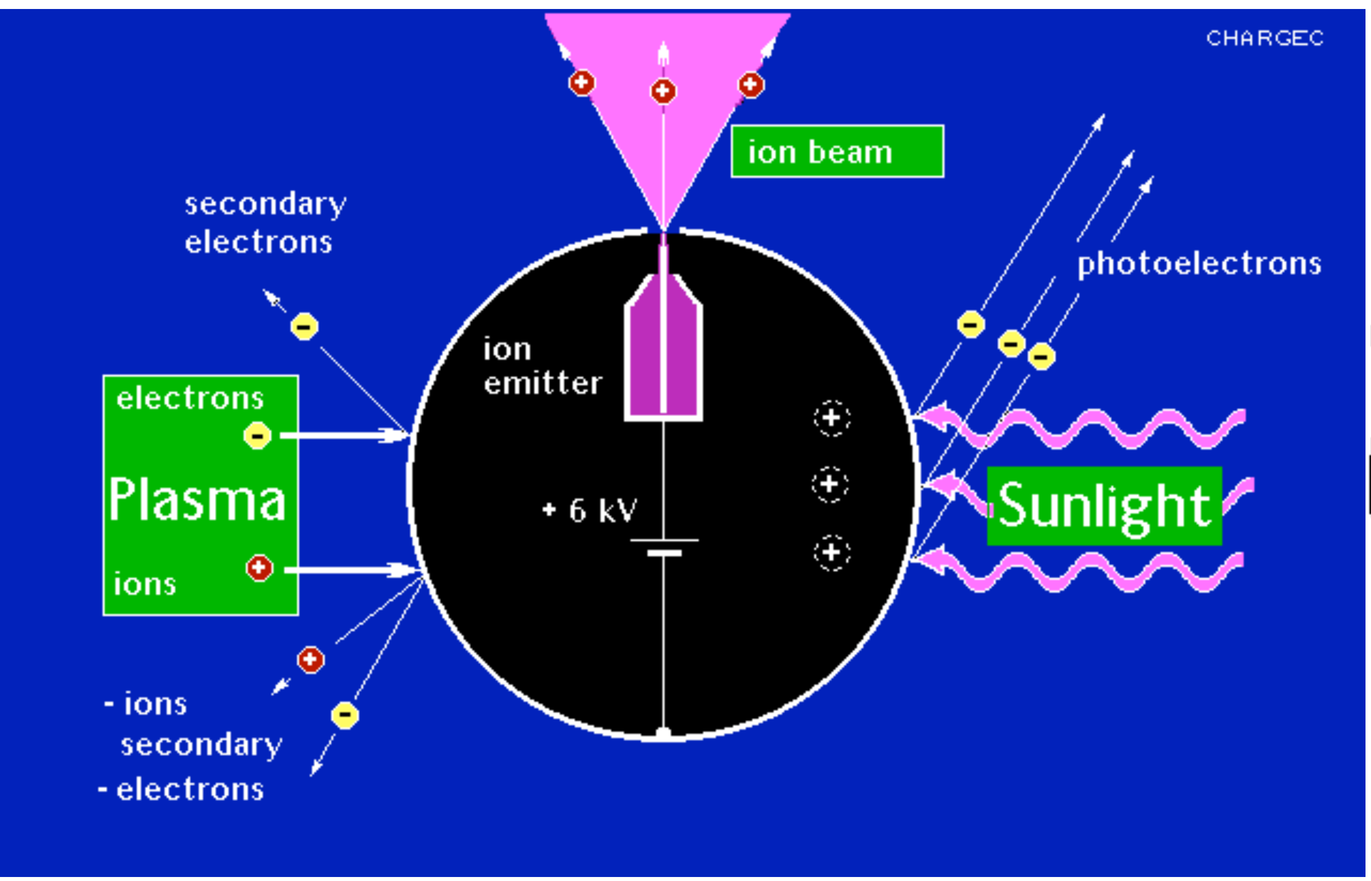
$$I_e + \sum_i I_i = 0 \rightarrow -\pi a^2 e n_o \sqrt{\frac{8\kappa T_e}{\pi m_e}} \exp\left[\frac{eV}{\kappa T_e}\right] + \pi a^2 q_i n_{io} U_s = 0 \rightarrow \boxed{V_f = -\frac{\kappa T_e}{2e} \ln\left(\frac{8\kappa T_e}{\pi m_e U_s^2}\right)}$$

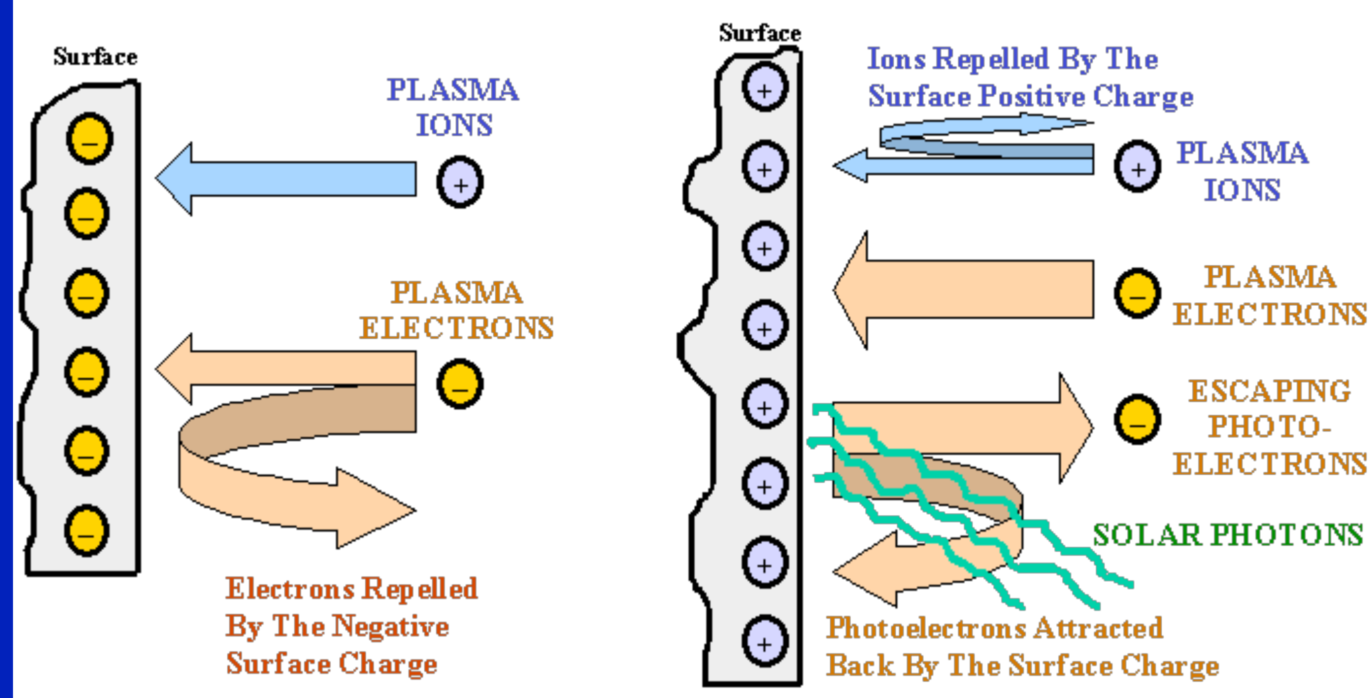
Definitions (cont.)





Surface charging



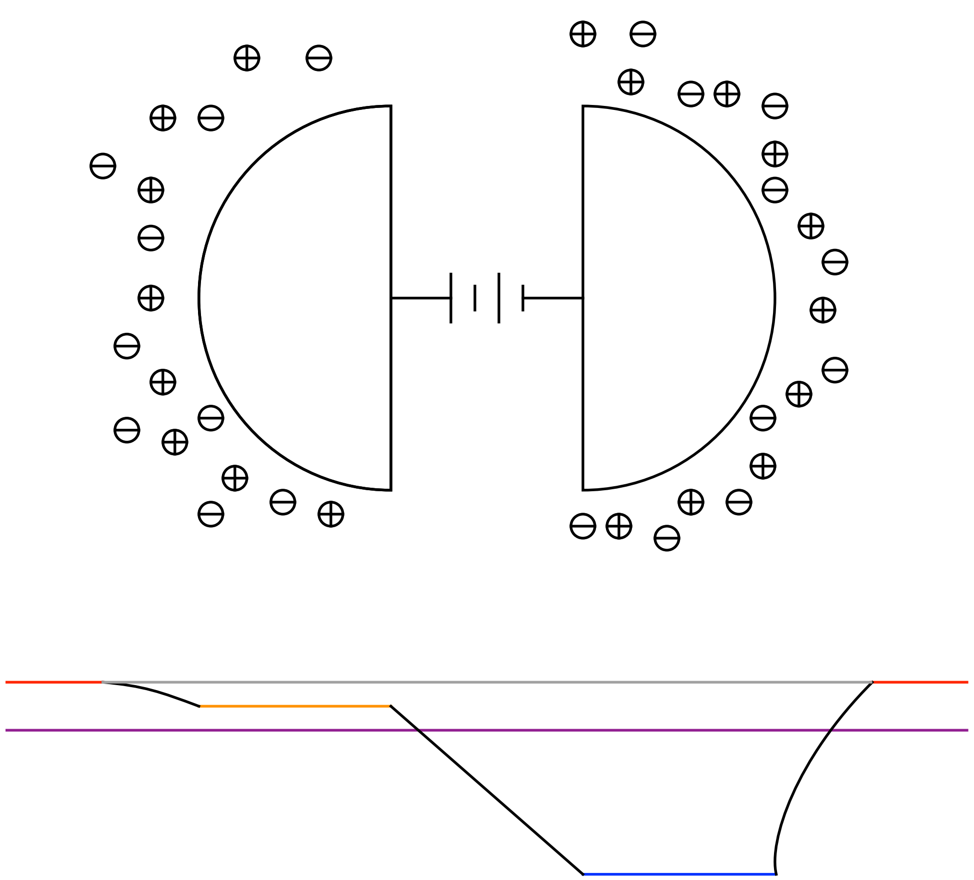


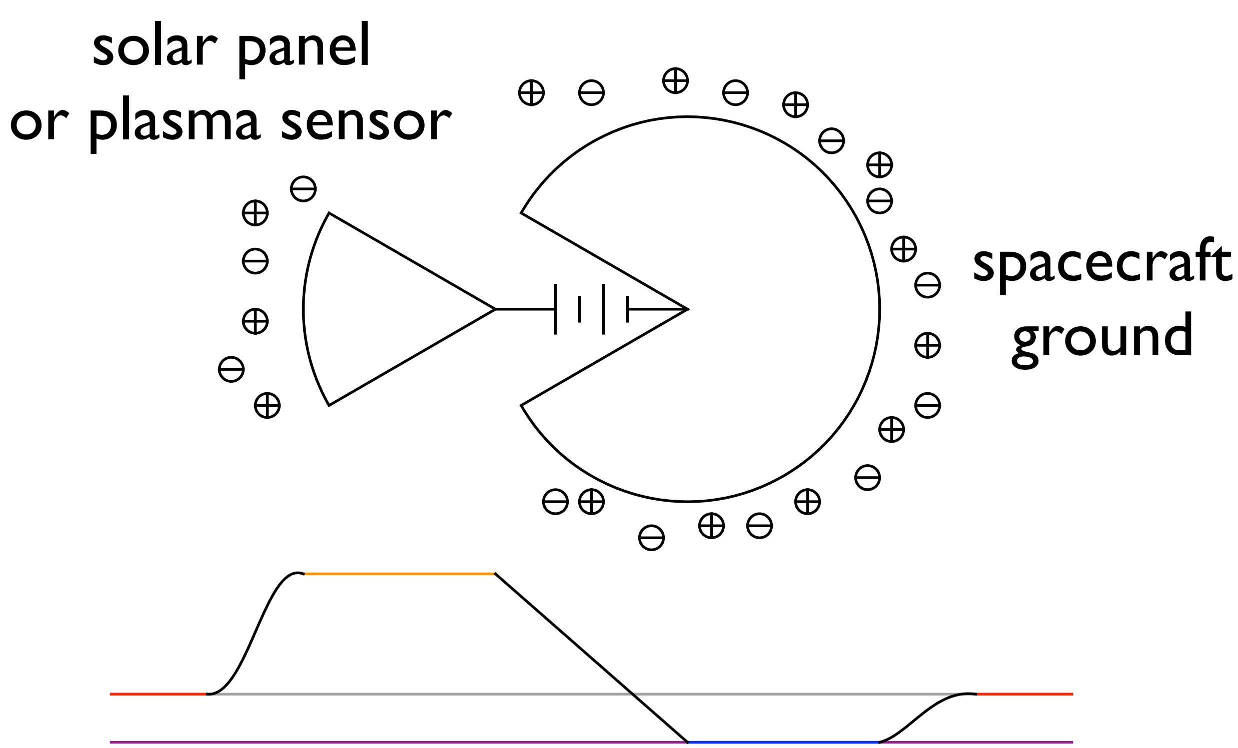
(a) Surface In Shadow

(b) Surface In Sunlight

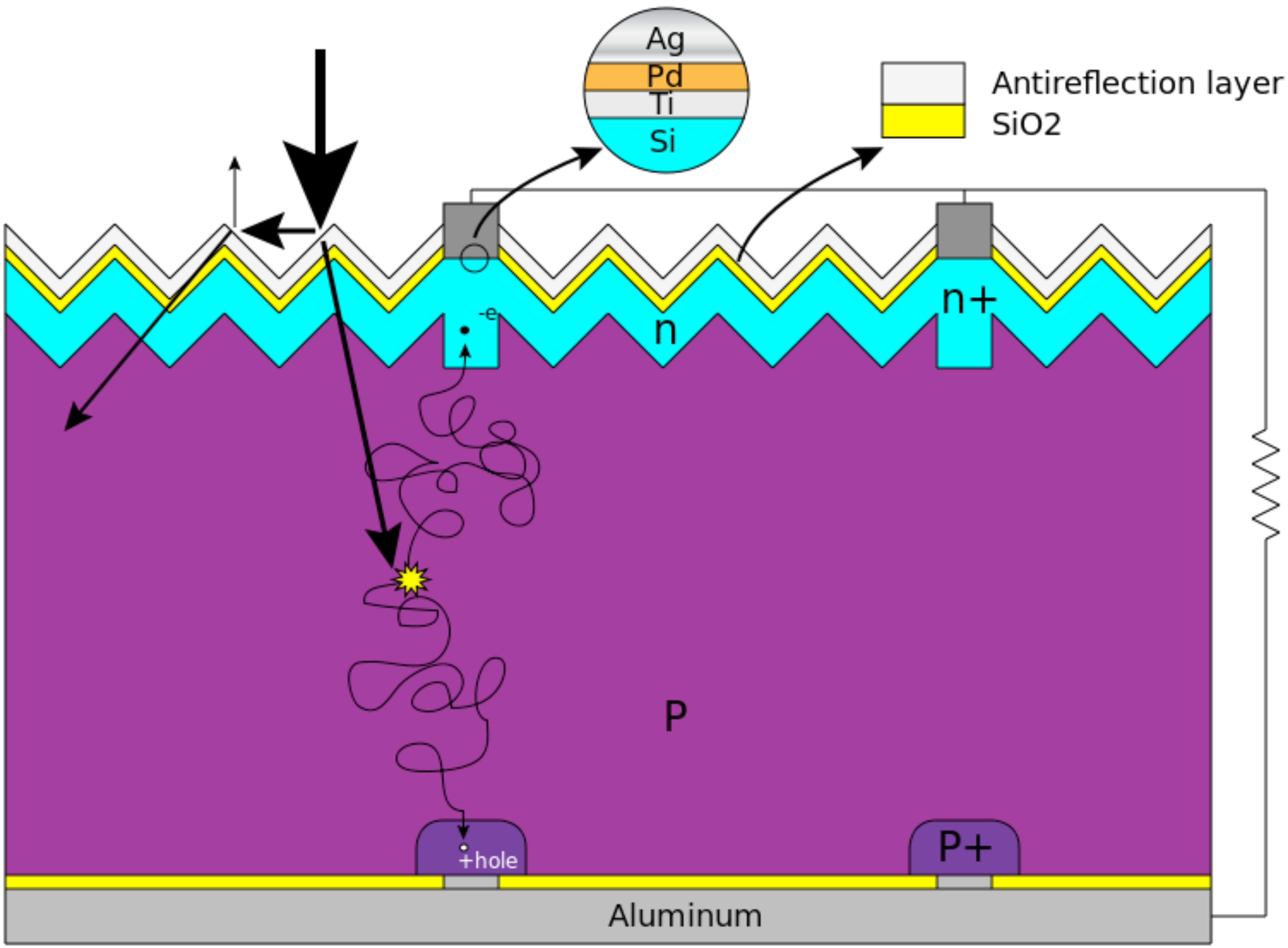
Definitions (cont.)

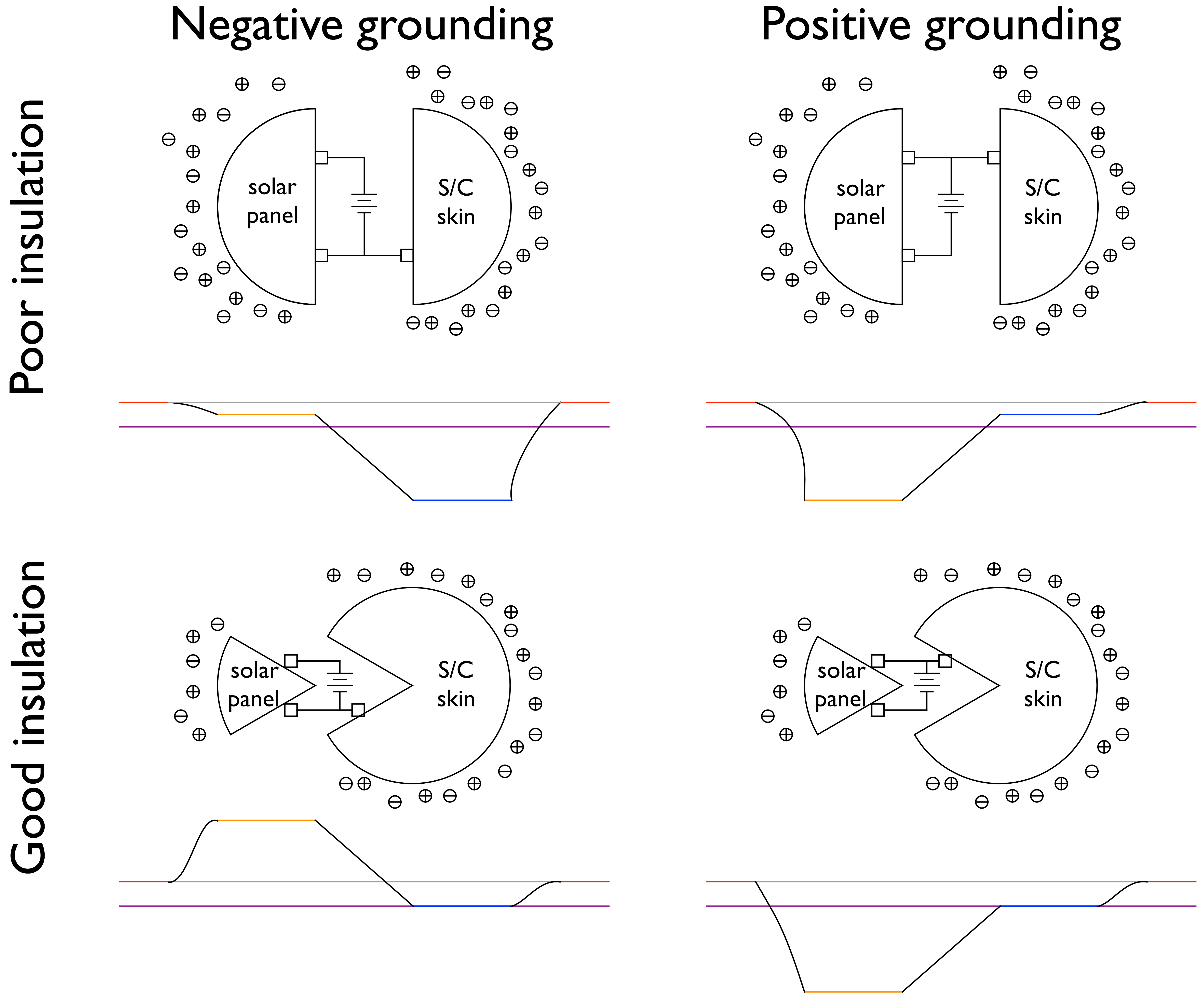
- **Spacecraft potential** (——): If a spacecraft (differential charging, the potential may be different on different surfaces) is immersed inside the plasma, the spacecraft potential will be kept at the floating potential to maintain the net current through the spacecraft to be zero in a steady state condition.



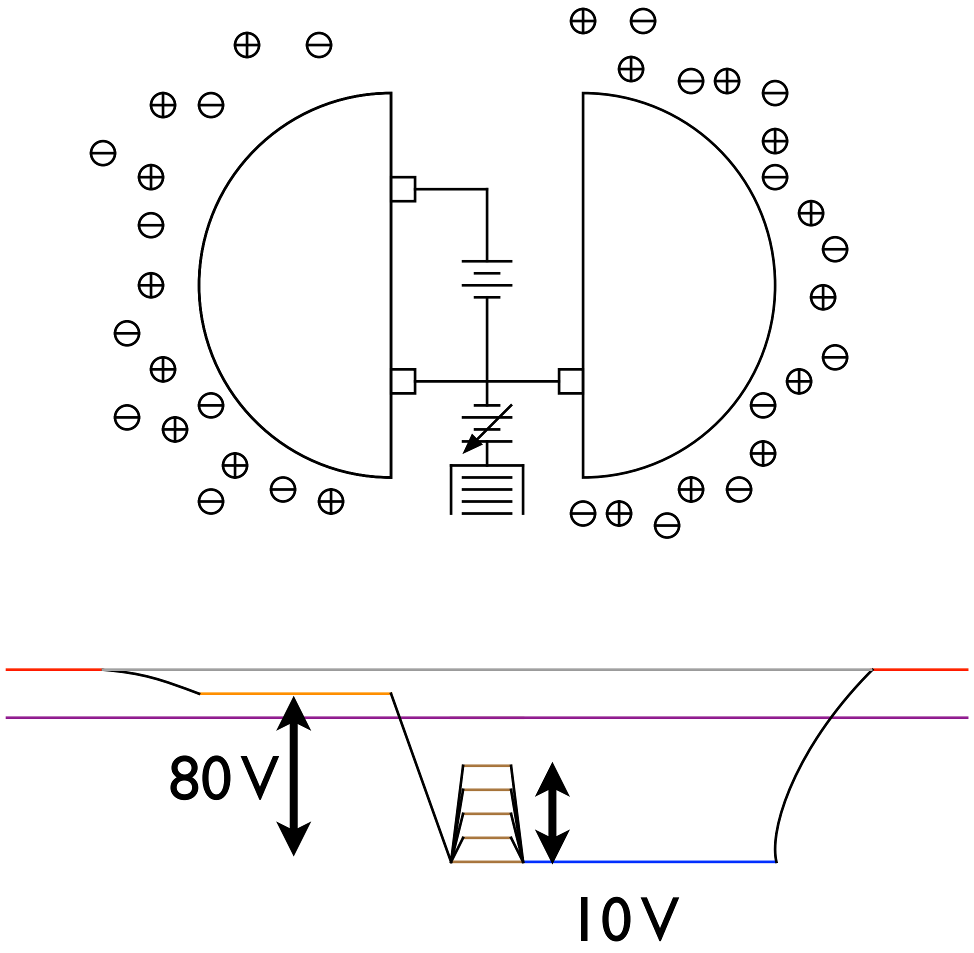


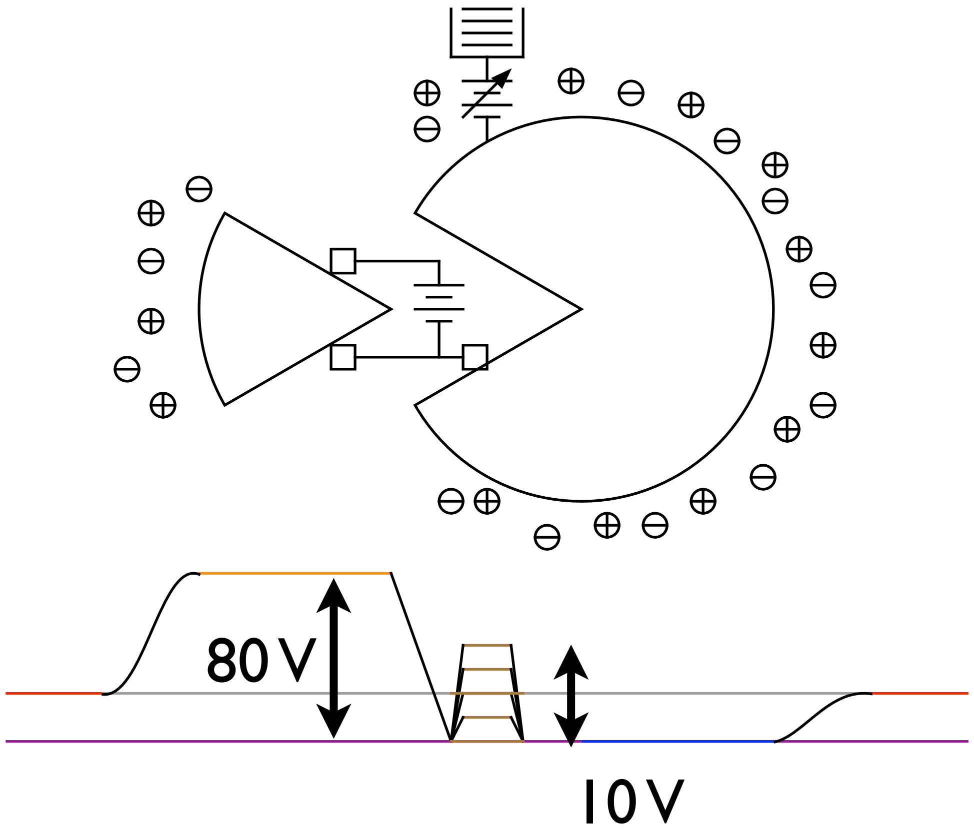
Solar cell - crystalline silicon




Negative grounding
Positive grounding
Poor insulation
Good insulation
solar panel
S/C skin
solar panel
S/C skin
solar panel
S/C skin
solar panel
S/C skin

Impact to in-situ plasma measurement

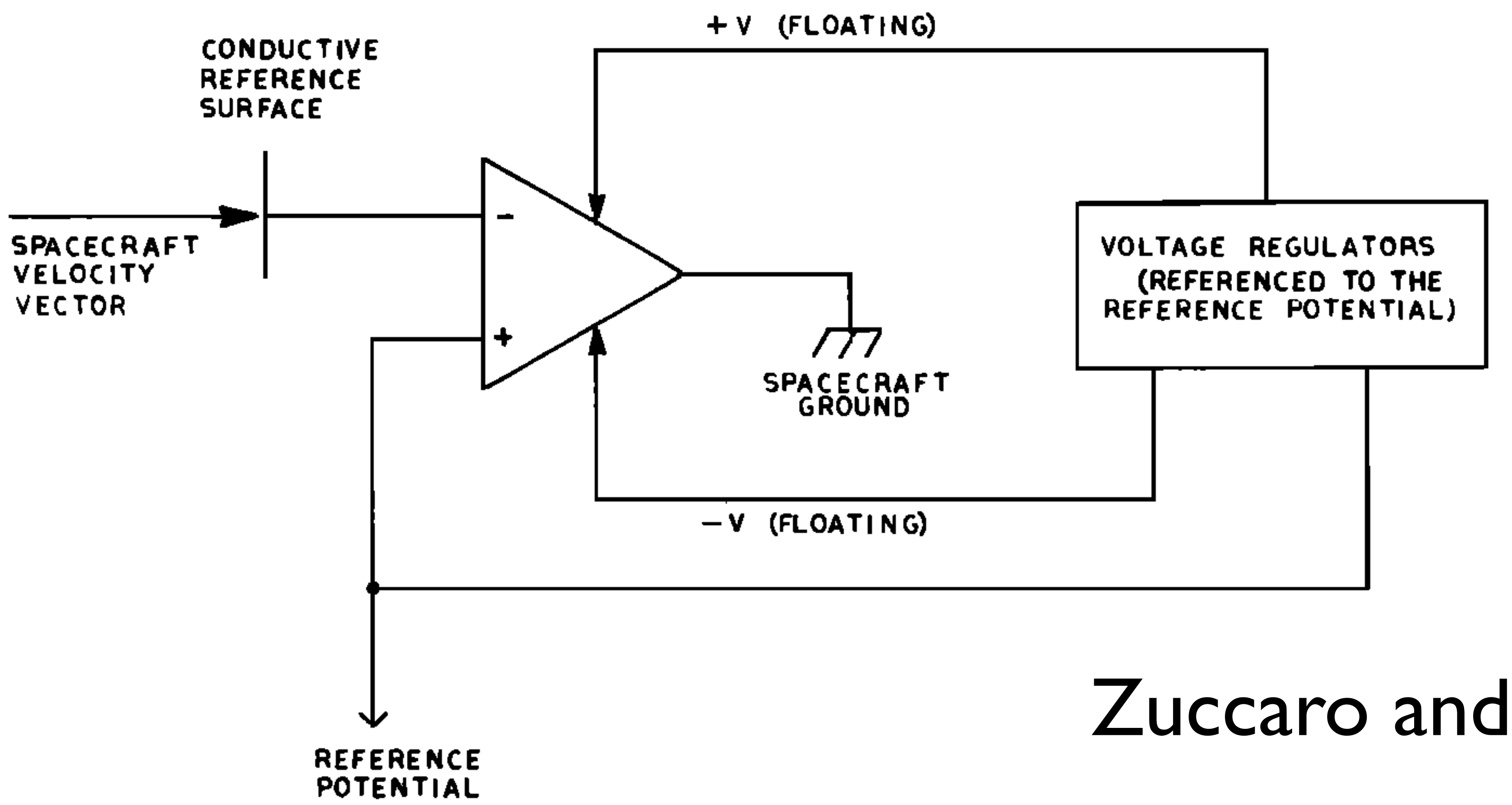




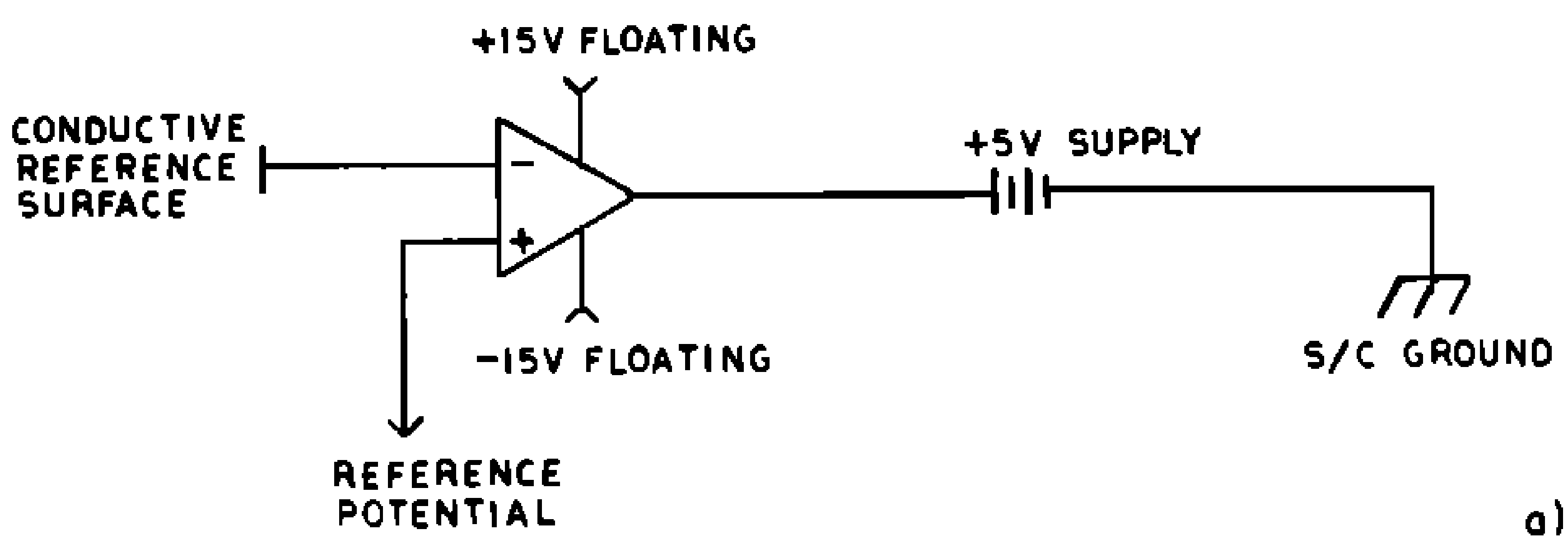
No sufficient voltage to block incoming plasma → No T_i

Solar panel may block low kinetic energy ions to sensor → No current

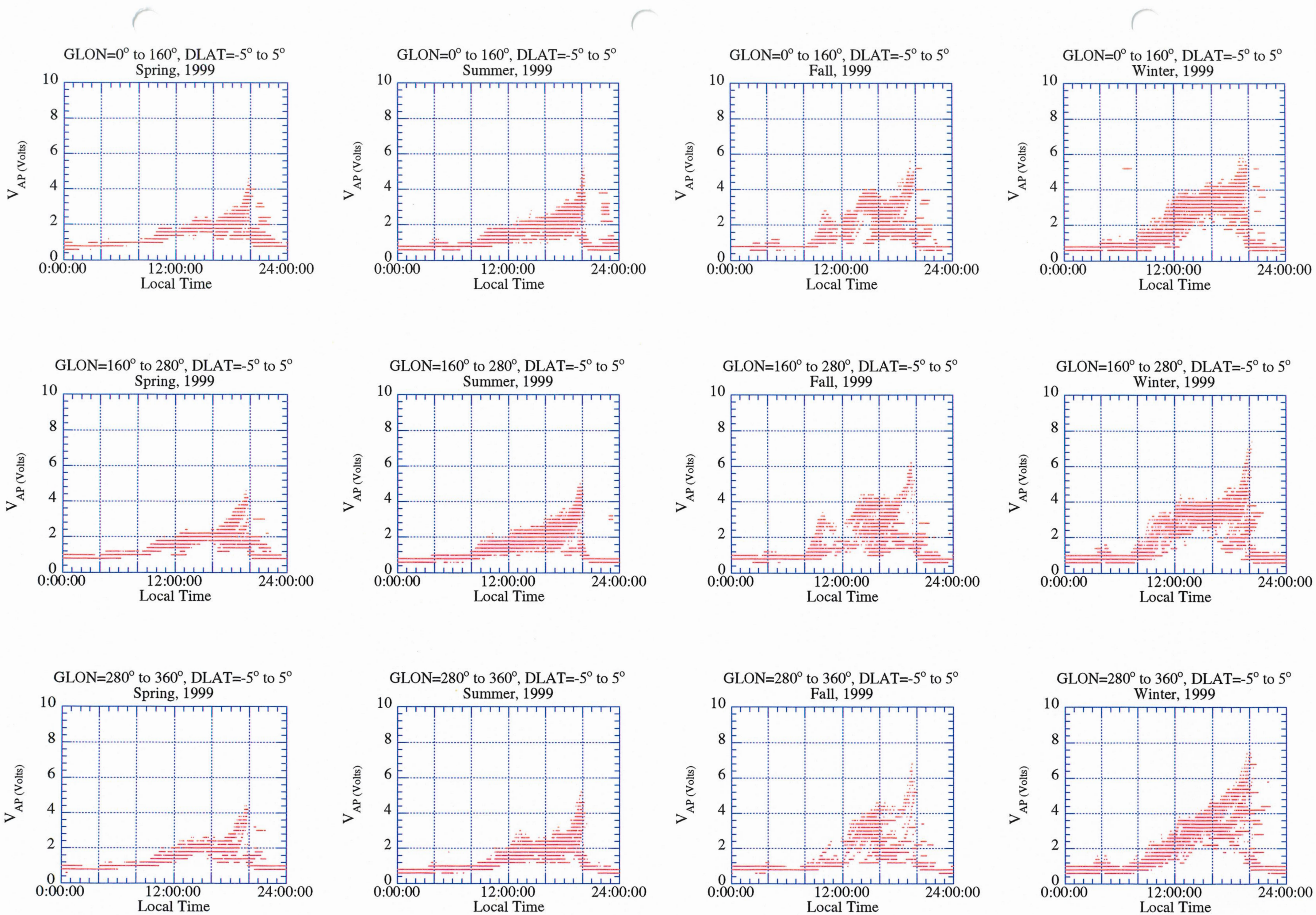
Senpot circuit



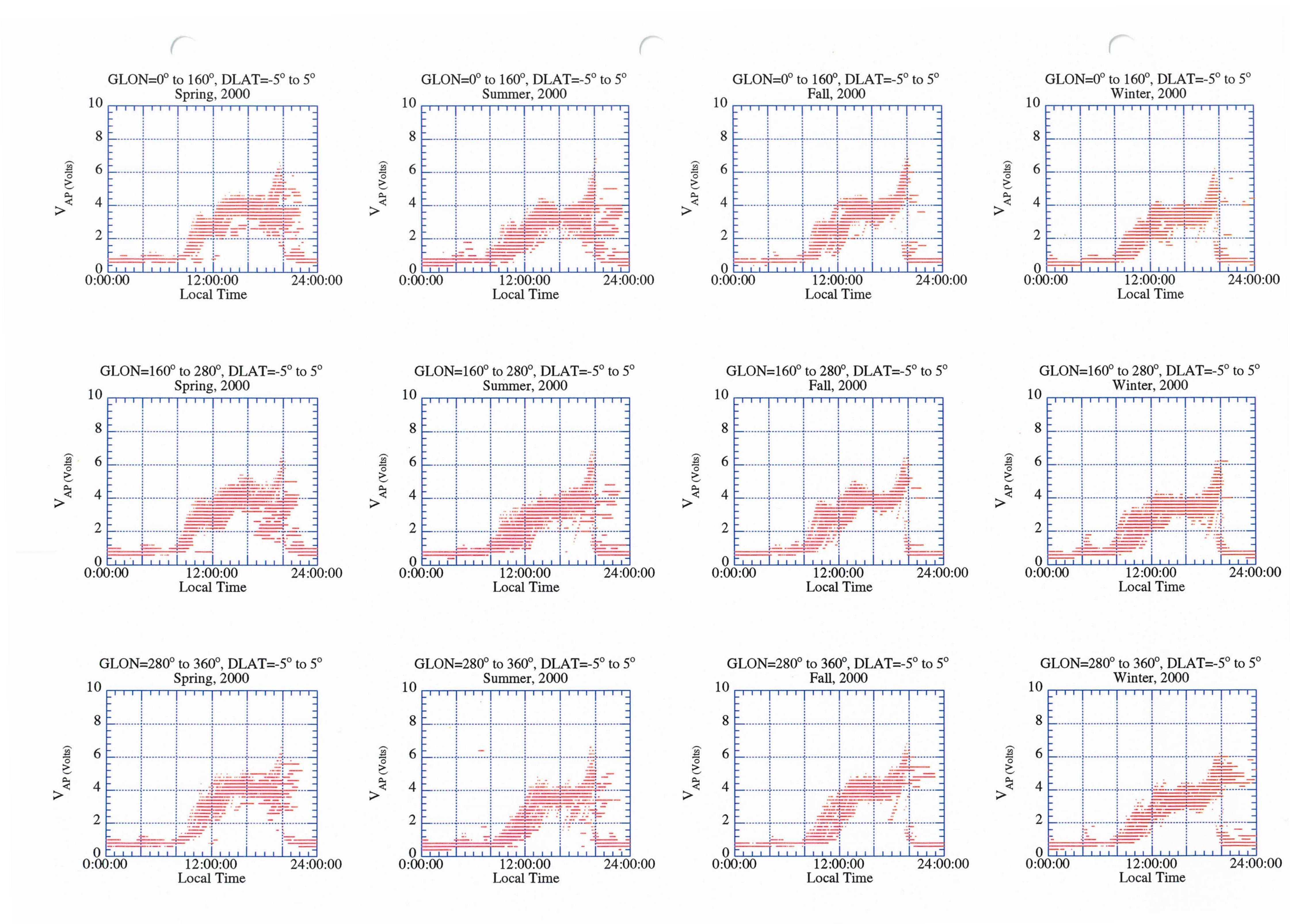
Zuccaro and Holt, 1982



V_{ap} measured by IPEI in 1999



V_{ap} measured by IPEI in 2000



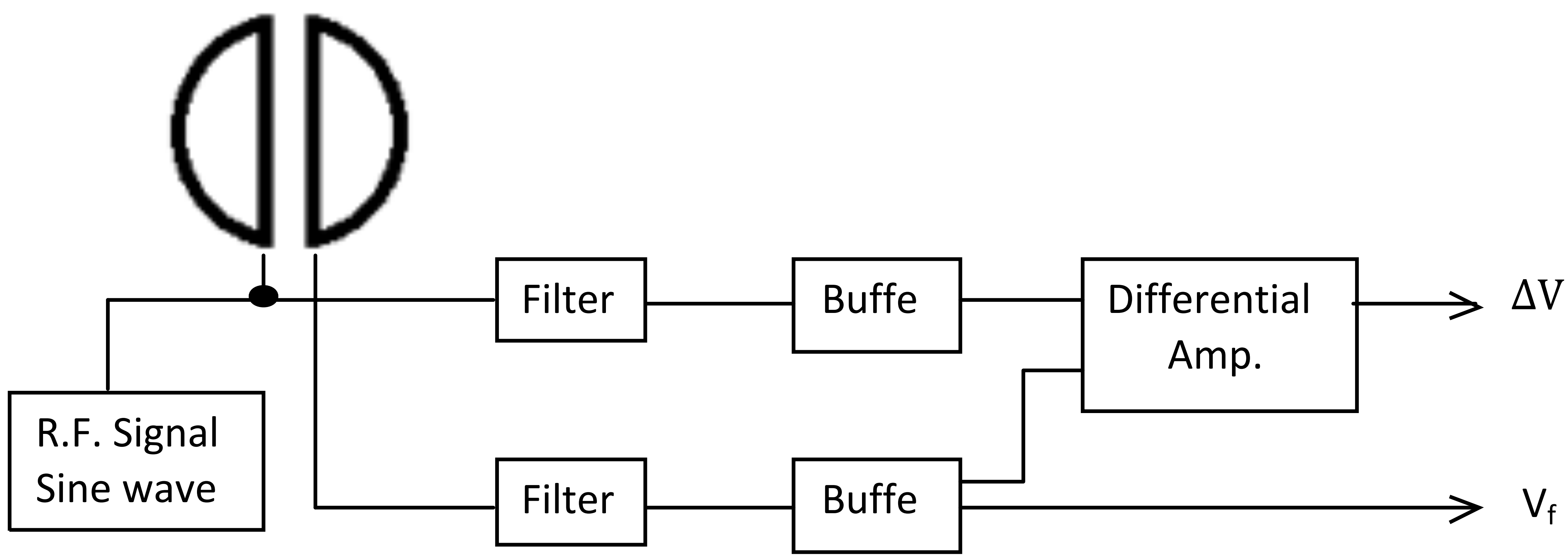
Summary

- Positive grounding can prevent the satellite ground far away from the plasma potential.
- Good insulation on the solar cell can maintain the satellite ground close to the plasma potential.
- Keep the solar panel away from the in-situ plasma sensor.
- In-situ plasma sensor cannot operate correctly under a negative grounding and poor insulation condition for an 80 V solar panel.
- Photoelectric effect is neglected for a LEO satellite (Kasha, 1969).

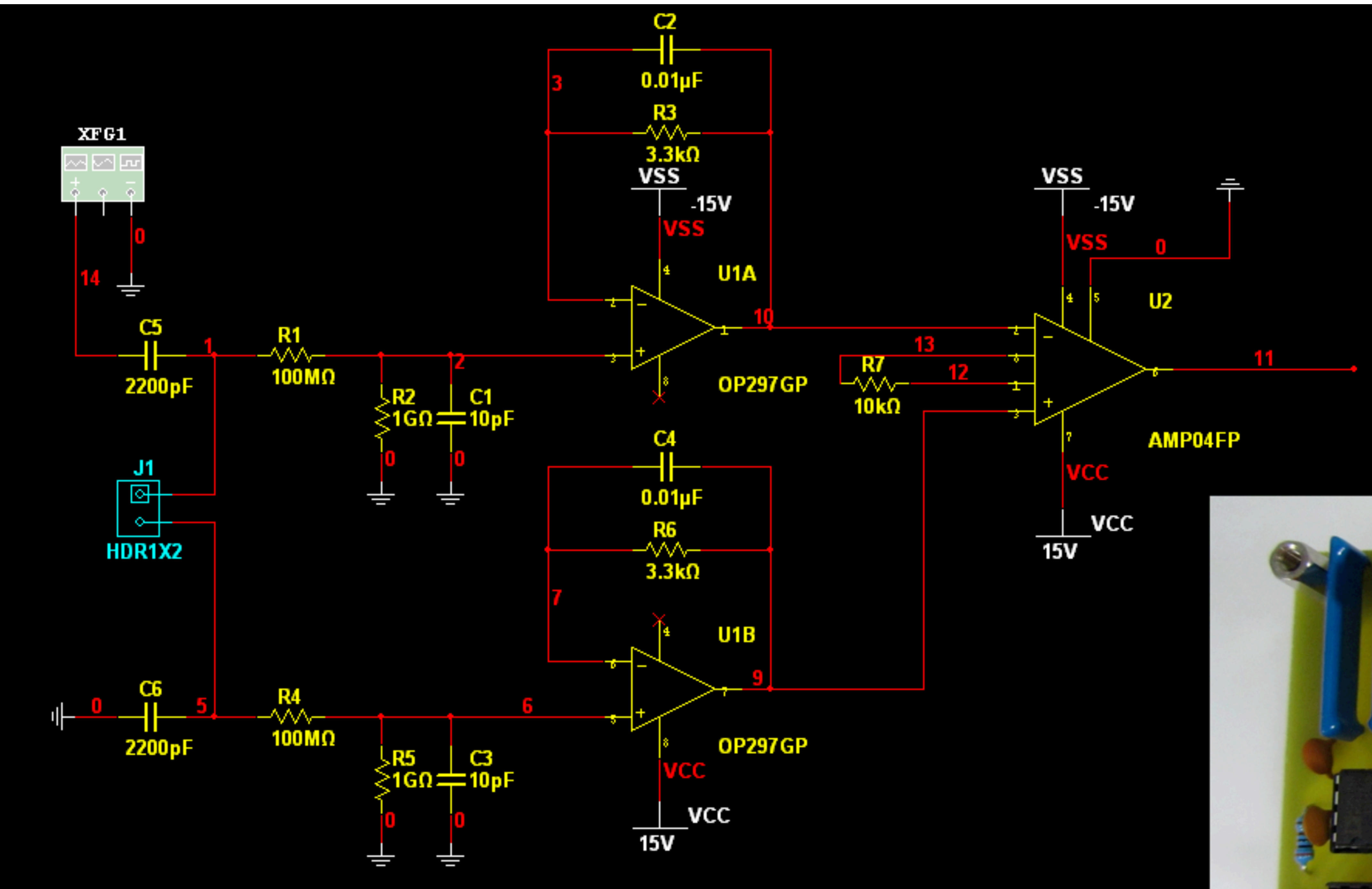
References

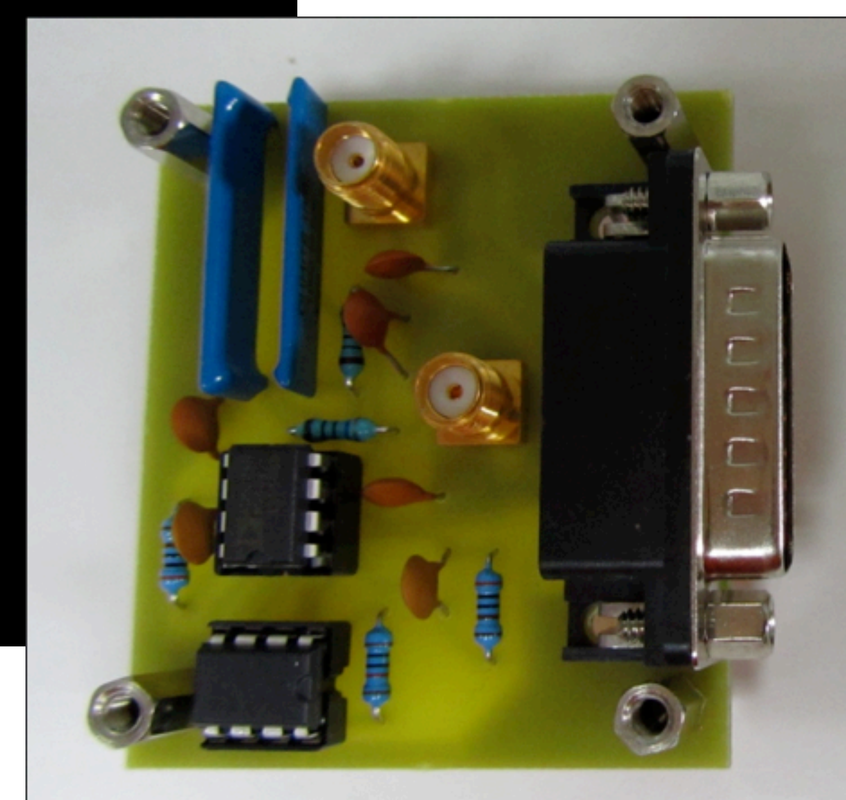
- Ferguson, D. C. and G. B. Hillard (2003), Low Earth Orbit Spacecraft Charging Design Guidelines, NASA/TP-2003-212287.
- Kasha, M. A. (1969), The Ionosphere and its Interaction with Satellites, Gordon and Breach, Science Publishers, Inc.
- NASA-STD-4005 (2007), Low Earth Orbit Spacecraft Charging Design Standard.
- NASA-HDBK-4006 (2007), Low Earth Orbit Spacecraft Charging Design Handbook.
- Zuccaro, D. R. and B. J. Holt (1982), A technique for establishing a reference potential on satellites in planetary ionospheres, J. Geophy. Res., 87, 8,327-8,329.

Functional diagram

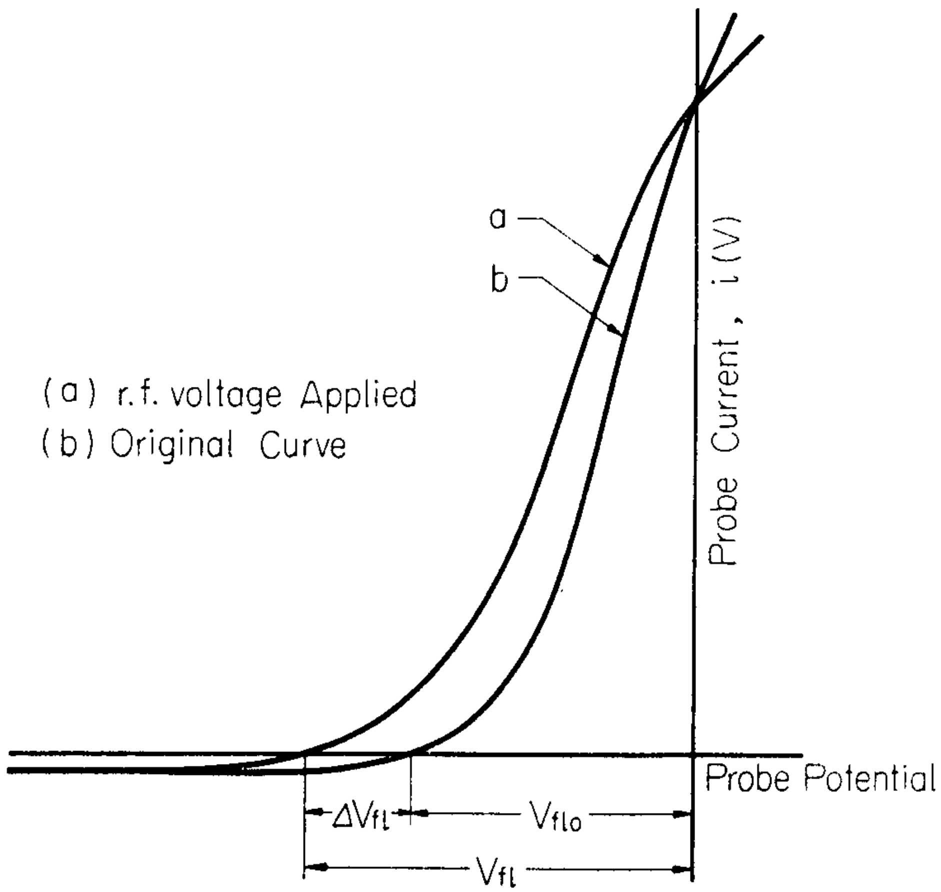


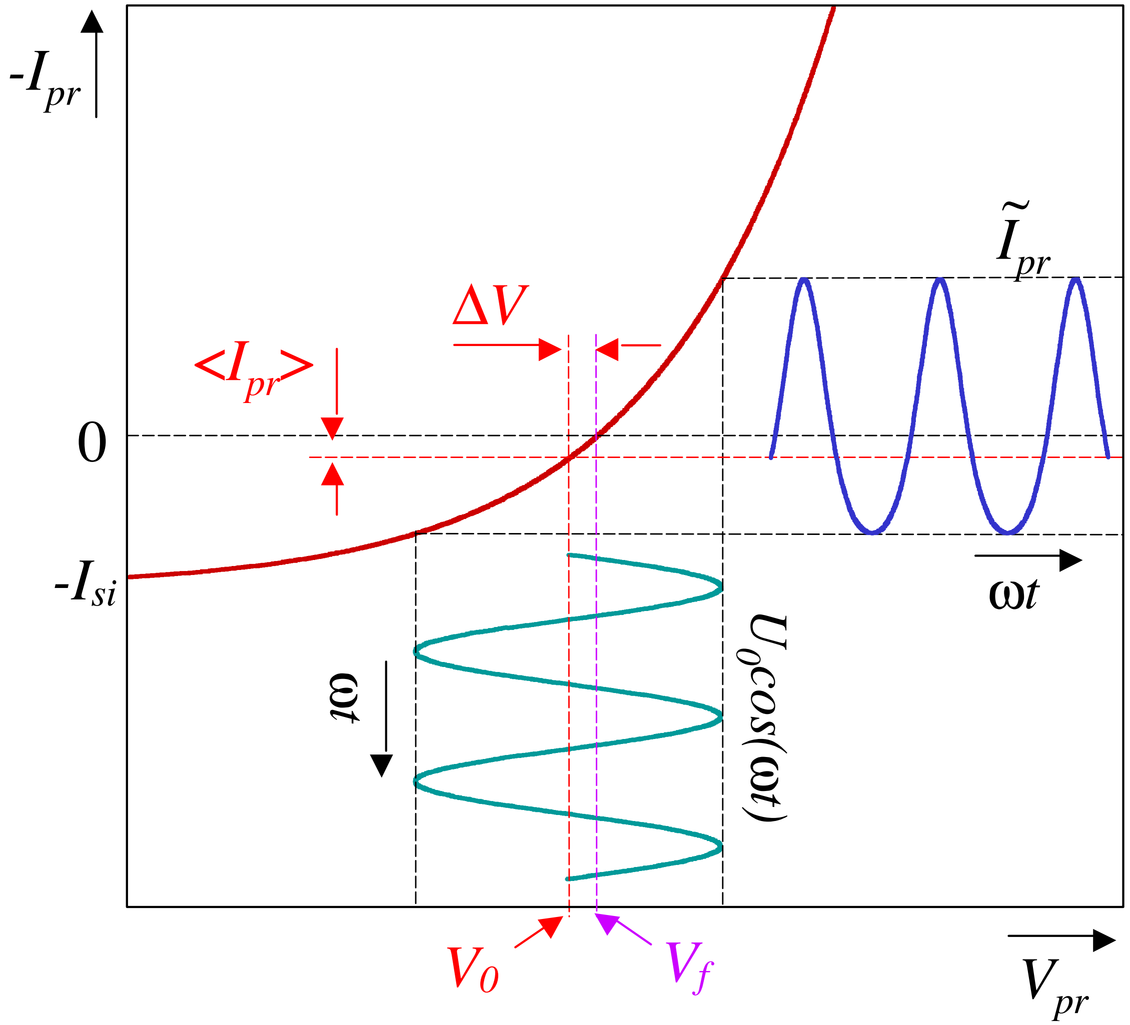
Schematic plot





I-V curves with/without RF voltage





Principle of the measurement

$$I = I_e + I_i$$

$$I_e = A(-e)n_o \exp\left(\frac{eV}{\kappa T_e}\right)\sqrt{\frac{8\kappa T_e}{\pi m_e}} = i_e \exp\left(\frac{eV}{\kappa T_e}\right) \text{ and } I_i = Aqn_o\upsilon_b$$

$$I_e = i_e \exp\left(\frac{eV(t)}{\kappa T_e}\right) = i_e \exp\left(\frac{e\left(V_0 + a\cos\omega t\right)}{\kappa T_e}\right)$$

$$= i_e \exp\left(\frac{eV_0}{\kappa T_e}\right)\exp\left(\frac{ea\cos\omega t}{\kappa T_e}\right) = i_e \exp\left(\frac{eV_0}{\kappa T_e}\right)\exp\left[i\left(-i\frac{ea}{\kappa T_e}\right)\cos\omega t\right]$$

$$= i_e \exp\left(\frac{eV_0}{\kappa T_e}\right)\left[I_0\left(-\frac{ea}{\kappa T_e}\right) + 2\sum_{n=1}^{n=\infty} i^n J_n\left(-i\frac{ea}{\kappa T_e}\right)\cos(n\omega t)\right]$$

where *a* is the amplitude and ω ($\ll \omega_{pe}$, but $> \omega_{pi}$) the frequency of a sinusoidal wave.

$J_n(z)$ is n-th Bessel function and $I_0(z)$ is the modified Bessel function of zeroth order. The Jacobi-Anger expansion:

$$e^{iz\cos\theta} = \sum_{n=-\infty}^{n=\infty} i^n J_n(z) e^{in\theta} = I_0(-iz) + 2\sum_{n=1}^{n=\infty} i^n J_n(z)\cos(n\theta),$$

$$I_\alpha(z) = (-1)^{-\alpha} J_\alpha(iz) \text{ and } J_{-n}(z) = (-1)^n J_n(z)$$

$$\left\langle \exp\left(\frac{ea\cos\omega t}{\kappa T_e}\right)\right\rangle = \left\langle I_0\left(-\frac{ea}{\kappa T_e}\right) + 2\sum_{n=1}^{n=\infty} i^n J_n\left(-i\frac{ea}{\kappa T_e}\right)\cos(n\omega t)\right\rangle$$

$$\sim I_0\left(-\frac{ea}{\kappa T_e}\right)$$

$$\langle I_e \rangle = i_e \exp\left(\frac{eV_0}{\kappa T_e}\right)\left\langle \exp\left(\frac{ea\cos\omega t}{\kappa T_e}\right)\right\rangle \sim i_e \exp\left(\frac{eV_0}{\kappa T_e}\right) I_0\left(-\frac{ea}{\kappa T_e}\right)$$

where the Bessel function $I_0(z)$ can be expanded as

$$I_0(z) = 1 + \frac{z^2}{2^2} + \frac{z^4}{2^2 \cdot 4^2} + \frac{z^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots$$

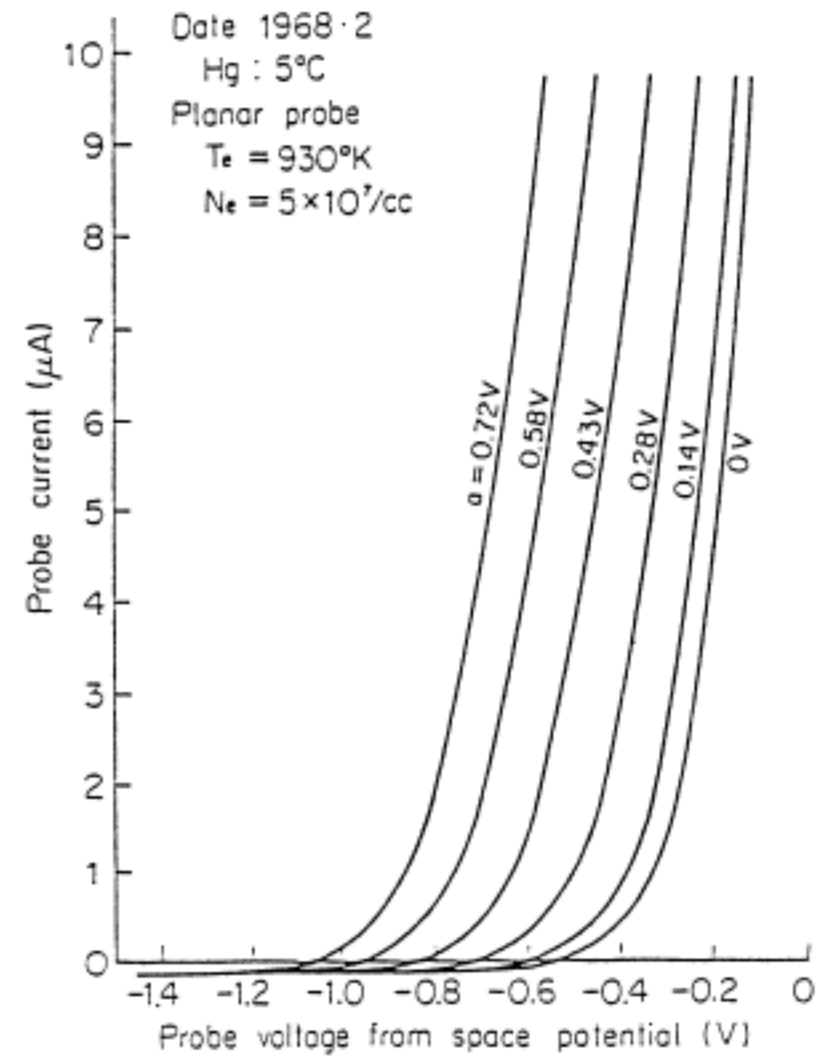


Fig. 1. Principle of the electron temperature probe. The V-I characteristic shifts when sine waves of an amplitude of 0.14, 0.28, 0.43, 0.58, and 0.72 V are superposed on the dc probe voltage. N_e and T_e are 5×10^7 cm^{-3} and 930 K respectively.

The floating potential

The floating potential without sinusoidal wave can be expressed as

$$I = I_e + I_i = i_e \exp\left(\frac{eV_f}{\kappa T_e}\right) + i_i = 0 \rightarrow V_f = -\frac{\kappa T_e}{e} \ln\left(\frac{i_e}{i_i}\right)$$

The floating potential with sinusoidal wave can be expressed as

$$I = I_e + I_i = i_e I_0\left(-\frac{ea}{\kappa T_e}\right) \exp\left(\frac{eV_f^a}{\kappa T_e}\right) + i_i = 0$$

$$\rightarrow V_f^a = -\frac{\kappa T_e}{e} \ln\left[\frac{i_e}{i_i} I_0\left(-\frac{ea}{\kappa T_e}\right)\right]$$

The shift between the floating potential and the original potential can be used to derived T_{e1}

$$\Delta V_f^a = V_f^a - V_f = -\frac{\kappa T_e}{e}\ln\left[\frac{i_e}{i_i} I_0\left(-\frac{ea}{\kappa T_e}\right)\right] - \left[-\frac{\kappa T_e}{e}\ln\left(\frac{i_e}{i_i}\right)\right]$$

$$= -\frac{\kappa T_e}{e}\ln\left[I_0\left(-\frac{ea}{\kappa T_e}\right)\right] \rightarrow T_{e1} = \frac{e\Delta V_f^a}{-\kappa \ln\left[I_0\left(-\frac{ea}{\kappa T_{e1}}\right)\right]}$$

When a sine wave with amplitude "$2a$" is applied to the electrode, the floating potential shift can be expressed and T_{e2} can be derived as

$$\Delta V_f^{2a} = V_f^{2a} - V_f = -\frac{\kappa T_e}{e}\ln\left[\frac{i_e}{i_i} I_0\left(-\frac{2ea}{\kappa T_e}\right)\right] - \left[-\frac{\kappa T_e}{e}\ln\left(\frac{i_e}{i_i}\right)\right]$$

$$= -\frac{\kappa T_e}{e}\ln\left[I_0\left(-\frac{2ea}{\kappa T_e}\right)\right] \rightarrow T_{e2} = \frac{e\Delta V_f^{2a}}{-\kappa \ln\left[I_0\left(-\frac{2ea}{\kappa T_{e2}}\right)\right]}$$

The ratio of the two floating potential shifts can be written and T_{e3} derived as

$$R = \frac{\Delta V_f^a}{\Delta V_f^{2a}} = \frac{-\frac{\kappa T_e}{e}\ln\left[I_0\left(-\frac{ea}{\kappa T_e}\right)\right]}{-\frac{\kappa T_e}{e}\ln\left[I_0\left(-\frac{2ea}{\kappa T_e}\right)\right]} = \frac{\ln\left[I_0\left(-\frac{ea}{\kappa T_e}\right)\right]}{\ln\left[I_0\left(-\frac{2ea}{\kappa T_e}\right)\right]} \rightarrow T_{e3}$$

T_e which are calculated from these three equations (T_{e1}, T_{e2}, and T_{e3}) should be equal.

Circuit of electron temperature probe

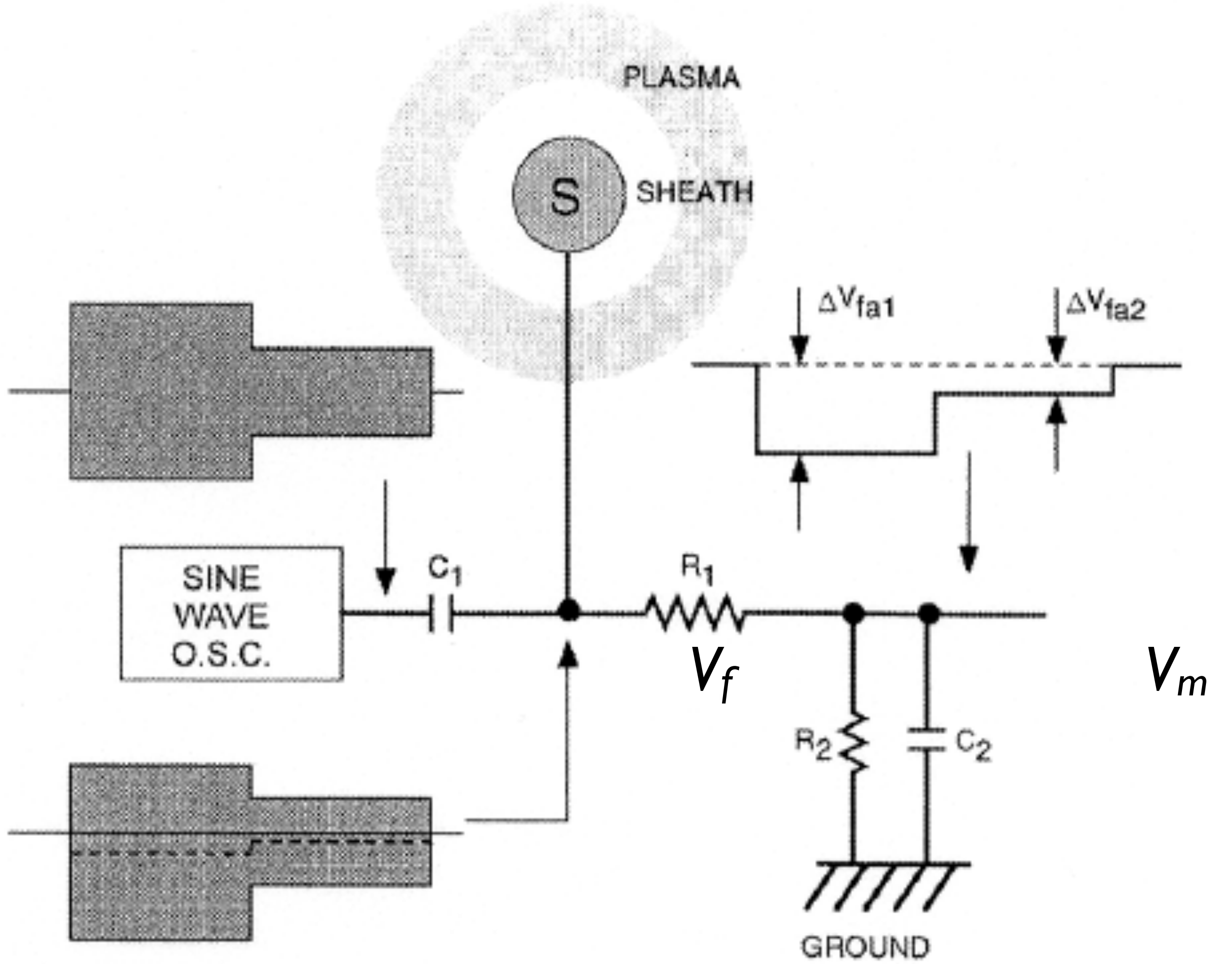
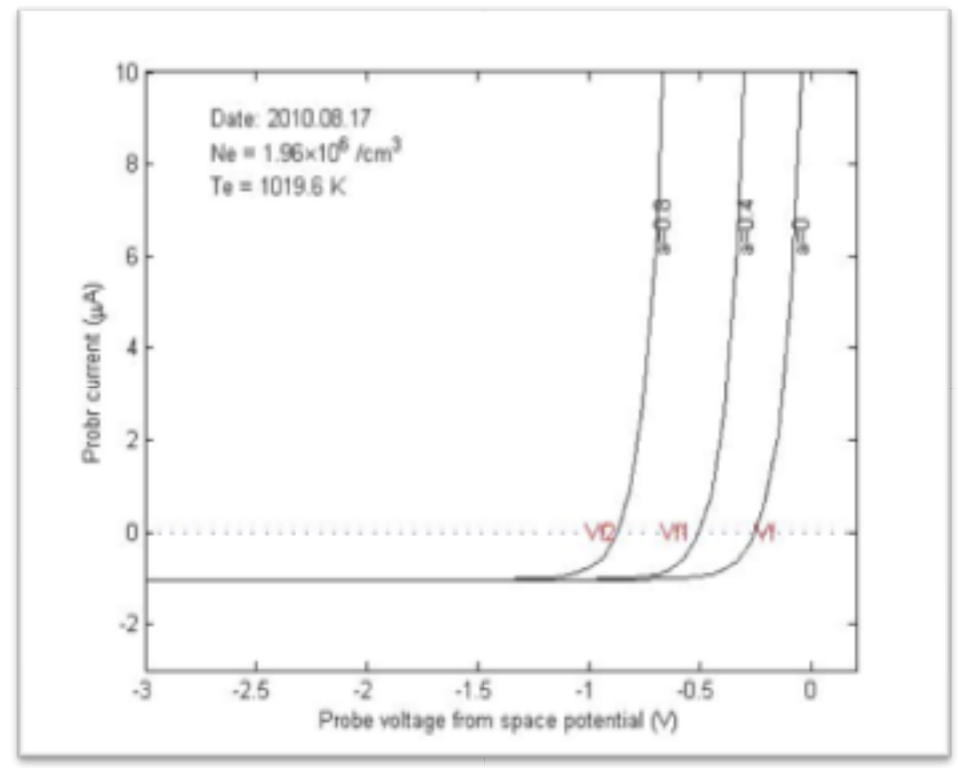


Fig. 2. Basic electronic circuit of the electron temperature probe. The sine wave is applied to the electrode through the capacitor C_1. It is thereby superposed on the floating potential shift generated by the application of this wave. The signal is then filtered by a C_2R_2 circuit yielding the pure floating potential. The sine wave is modulated at three different amplitudes (500 mV, 250 mV and 0 V, shown in gray).



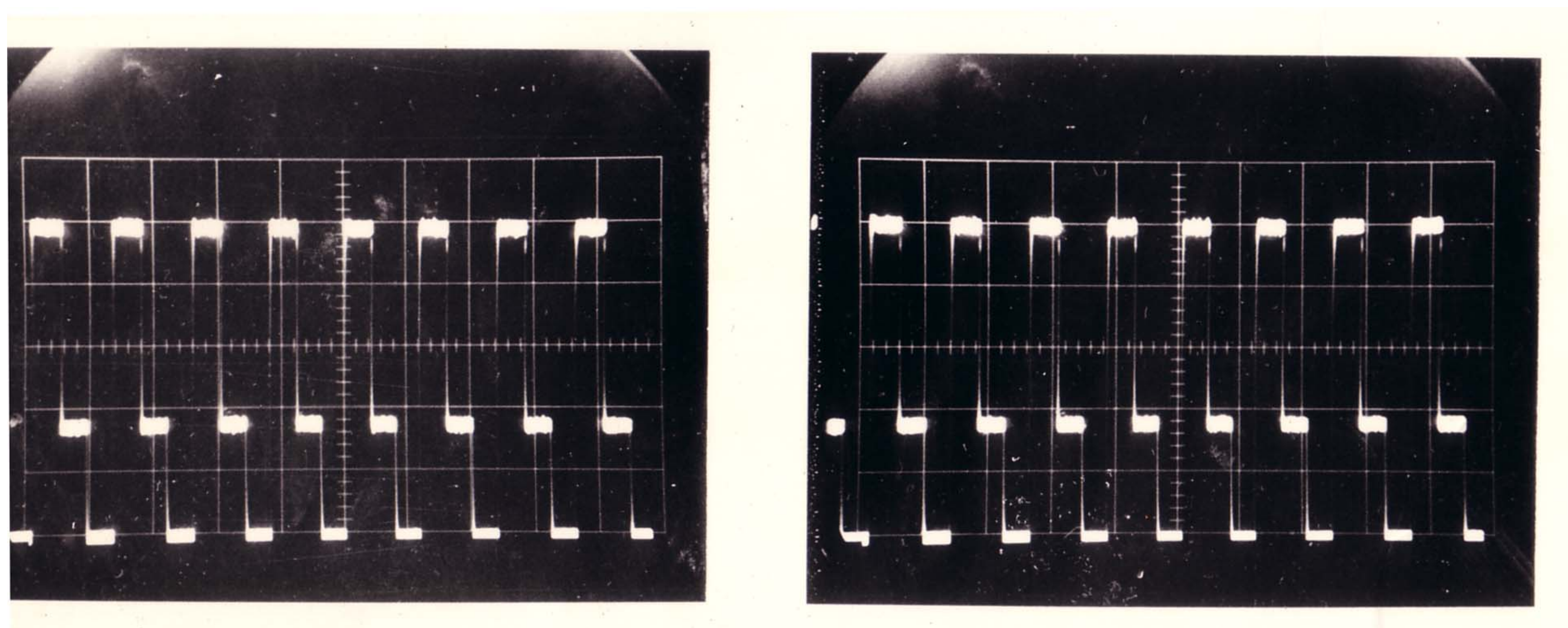
$$\frac{V_f - V_m}{R_1} = \frac{V_m - 0}{R_2} \rightarrow V_f = R_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_m = \frac{R_1 + R_2}{R_2}V_m$$

$$\rightarrow V_m = \frac{R_2}{R_1 + R_2}V_f \rightarrow \Delta V_m = G\left(V_{m1} - V_{m2}\right)$$

$$= G\left[\frac{R_2}{R_1 + R_2}\left(V_f + \Delta V_f^a\right) - \frac{R_2}{R_1 + R_2}V_f\right] = G\Delta V_f^a \frac{R_2}{R_1 + R_2}$$

Advantage of ETP

Clean probe　　　　Unclean probe



Effect of electrode contamination can be minimized.